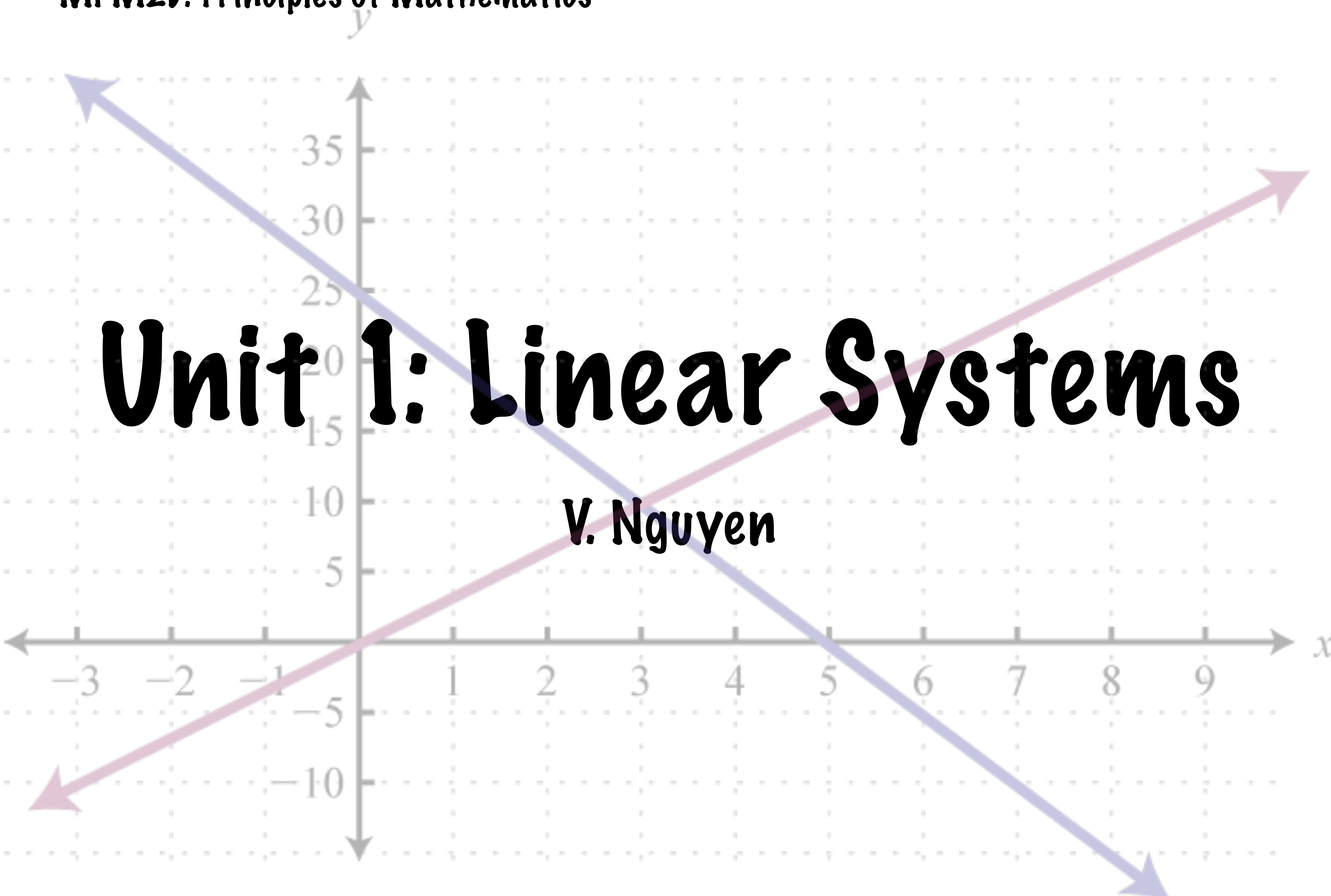


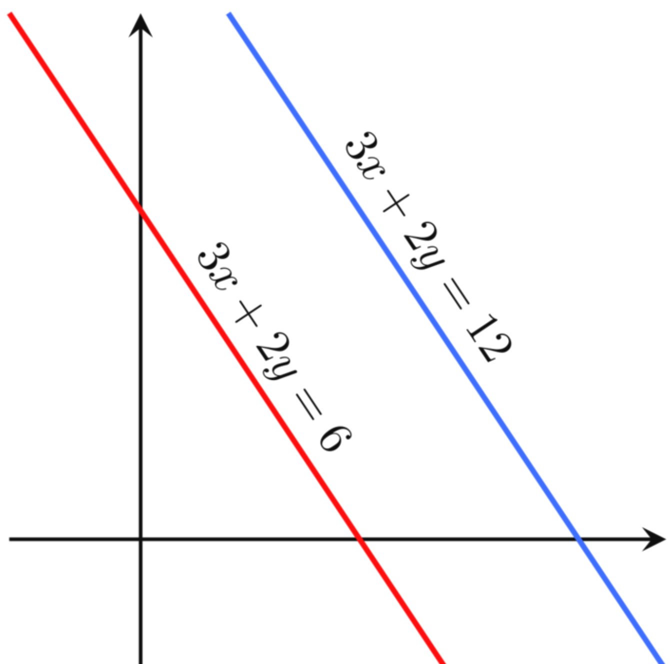
Unit 1: Linear Systems

V. Nguyen

Solving Linear Systems

Solving by Graphing

- **Goal:** Investigate and solve problems that involve systems of linear equations, or simply linear systems graphically.



Graphing Linear Systems

Systems of Equations: A series of two or more equations with the same set of unknowns.

e.g. ______________________ ______________________

______________________ ______________________

____

Linear systems: A system of equations in which all equations represent linear relations.

e.g. ______________________ ______________________

______________________ ______________________

Solving Linear Systems Graphically

Example 1: Graph the linear system defined by the two equations

$$y = 2x - 4$$
$$y = -x + 8$$

Recall: $y = mx + b$

Solving Linear Systems Graphically

In the previous example, the two lines intersect at (,   ).
This means that (,   ) is a ___________________________ for both equations.

In other words:

Basically to solve a linear system is to find the values of <u>x</u> and <u>y</u>, such that they satisfy both equations.

<u>**Solution of a Linear System:**</u> Graphically, the solution to a linear system is the <u>point of intersection (POI)</u> of the two lines.

Solving Linear Systems Graphically

Example 2: Graph the linear system defined by the two equations

$$x - 2y = -6$$

$$2x + y = 8$$

Solving Linear Systems Graphically

Example 3: Graph the linear system defined by the two equations

$$y = -\frac{1}{2}x - 2$$
$$2x + 4y = 12$$

Solving Linear Systems Graphically

Example 3: Graph the linear system defined by the two equations

$$y = -\frac{1}{2}x - 2$$
$$2x + 4y = 12$$

Number of Solutions

In the previous example, there was NO SOLUTION to the linear system because the two lines were ______________________.

A linear system with no solution is ____________________.

Generally, a linear system may have:

* 1 unique solution (if the two lines intersect)
* 0 solutions (if the two lines are parallel and distinct)
* infinite solutions if the two lines are coincident)

<u>Tip:</u> Two lines are parallel if they have the same slope!

Number of Solutions

Example 4: Determine the number of solutions of the linear system given by:

$$y = \frac{3}{2}x + 1$$
$$3x - 2y = 10$$

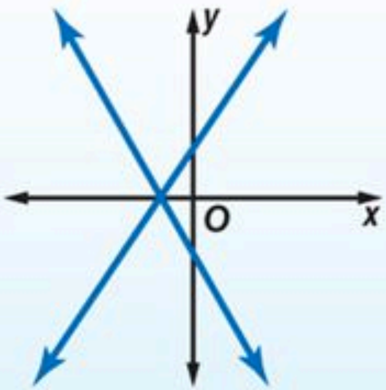
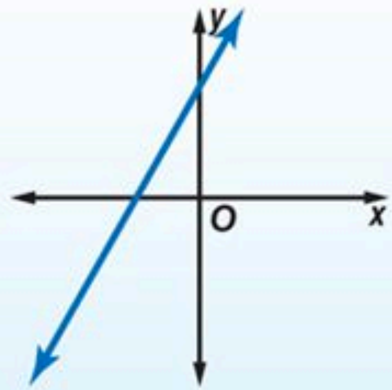
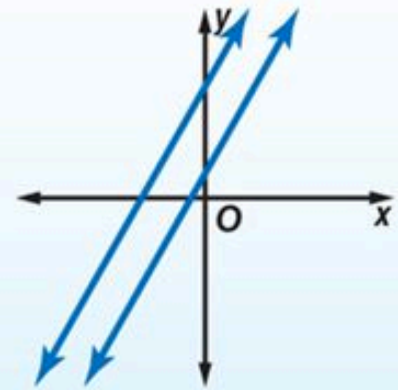
Number of Solutions

Example 5: Determine the number of solutions of the linear system given by:

$$y = \frac{5}{2}x + 3$$
$$5x - 2y = -6$$

Concept Summary

Characteristics of Linear Systems

Consistent and Independent	Consistent and Dependent	Inconsistent
		
intersecting lines; one solution	same line; infinitely many solutions	parallel lines; no solution

Solving Linear Systems

Solving by Substitution

* **Goal:** Investigate and solve problems that involve systems of linear equations, or simply linear systems using algebra.

$y = x + 1 \qquad y = -x$

$$\begin{array}{r} y = x + 1 \\ + \quad y = -x \\ \hline 2y = 1 \end{array}$$

Eliminate X

$$y = \frac{1}{2}$$

$\downarrow$

$$y = -x$$

Substitute

$$-\frac{1}{2} = x$$

Solution$\left(-\frac{1}{2}, \frac{1}{2}\right)$

Limitations with Graphing:

Graphing is a useful technique, but it has several limitations when it comes to solving linear systems graphically.

- It is time-consuming: figuring out scale for graph, locate y-intercept, slope.

- Points of intersection may be hard to read accurately.

Solving Linear Systems Using Substitution

Example 1: Solve using Substitution

$$y = 3x - 7$$

$$y = -x + 5$$

Solving Linear Systems Using Substitution

Example 2: Solve the linear system $y = 5x - 11$ and $y = 3x + 5$

Solving Linear Systems Using Substitution

Example 3: Solve the linear system $y = 2x + 5$ and $y = -4x + 12$

Solving Linear Systems Using Substitution

Example 4: Solve the linear system $y = 7x + 3$ and $y = 7x - 8$

Solving Linear Systems Using Substitution

Example 5: Solve the linear system $y = -\frac{1}{3}x - \frac{1}{2}$ and $y = -\frac{1}{6}x - \frac{9}{4}$

Solving Linear Systems Using Substitution

Solving Linear Systems Using Substitution (II)

What if the equations is not in slope-intercept form? For example:

$$y = 2x - 10$$

$$3x + y = 25$$

You can see that y is equivalent to 2x - 10, so it can be substituted into the second equation.

Solving Linear Systems Using Substitution (II)

Example 1: Solve the linear system $y = -4x + 3$ and $7x - 5y = -15$

Solving Linear Systems Using Substitution (II)

Example 2: Solve the linear system $y = 5x + 3$ and $5x - y = 8$

Solving Linear Systems Using Substitution (II)

Example 3: Solve the linear system $2x + 3y = 3$ and $-4x - 3y = 15$.

Solving Linear Systems

Solving by Elimination

- **Goal:** Investigate and solve problems that involve systems of linear equations, or simply linear systems using algebra.

$y = x + 1 \qquad y = -x$

$$\begin{array}{r} y = x + 1 \\ + \quad y = -x \\ \hline 2y = 1 \\ y = \frac{1}{2} \end{array}$$

Eliminate X

$y = -x$

Substitute

$-\frac{1}{2} = x$

Solution $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Solving Linear Systems Using Elimination

Example 1: Solve using Elimination

$$9x - 5y = -24$$

$$9x - 2y = -15$$

Solving Linear Systems Using Elimination

Example 2: Solve the linear system $3x + 8y = 10$ and $5x - 8y = 6$.

* Eliminate terms based on their **SIGNS** (i.e. subtract if coefficients of similar term if they have same sign, add if the signs are different just like this eg).

Solving Linear Systems Using Elimination

Example 3: Solve the linear system $4x + 3y = 7$ and $4x + 3y = 2$.

Solving Linear Systems

Solving by Elimination (II)

* **Goal:** Investigate and solve problems that involve systems of linear equations, or simply linear systems using algebra.

$y = x + 1 \qquad y = -x$

$$\begin{aligned} y &= x + 1 \\ + \quad y &= -x \\ \hline 2y &= 1 \\ y &= \frac{1}{2} \end{aligned}$$

Eliminate X

$$y = -x$$

Substitute

$$-\frac{1}{2} = x$$

Solution$\left(-\frac{1}{2}, \frac{1}{2}\right)$

Solving Linear Systems Using Elimination (II)

All of the previous examples in Elimination (I) involved terms with similar coefficients. Elimination can also be used when there are no terms with similar coefficients.

e.g.
$$x + 3y = 7$$
$$2x - 5y = 10$$

Solving Linear Systems Using Elimination (II)

Example 1: Solve the linear system $3x + 2y = 18$ and $9x + 4y = 60$.

Solving Linear Systems Using Elimination (II)

Example 2: Solve the linear system $10x - 25y = 11$ and $15x + 5y = 8$.

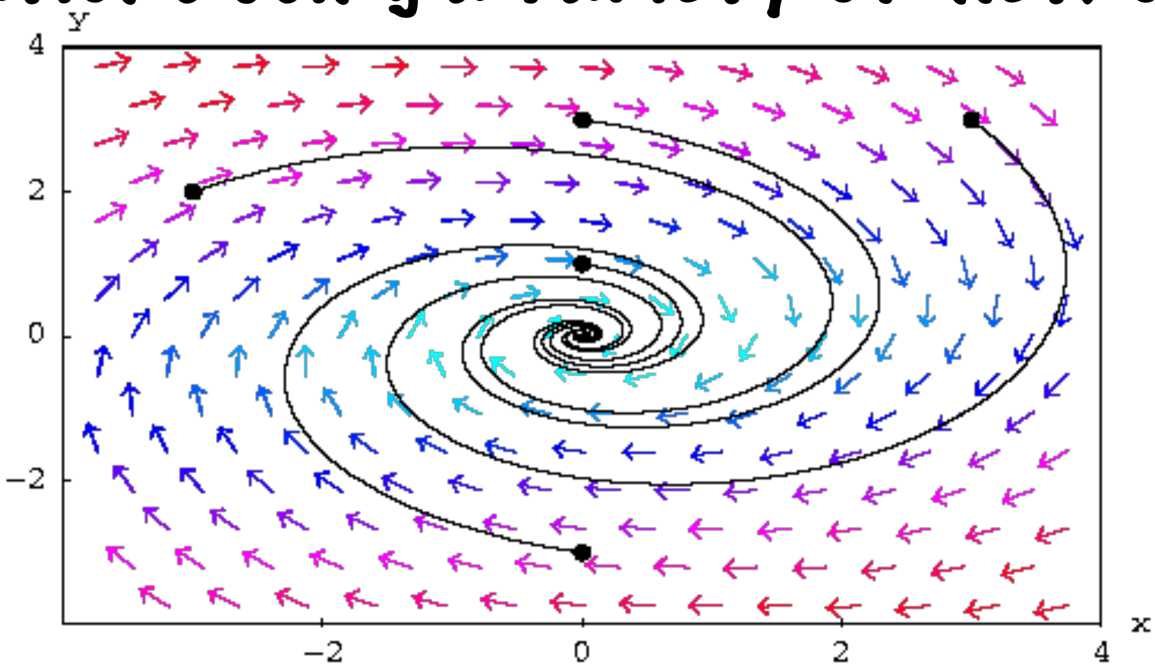
Solving Linear Systems Using Elimination (II)

Example 3: Solve the linear system $3x - 10y = -27$ and $6x + 15y = 51$.

Solving Linear Systems

Applications

* **Goal:** Investigate and solve application problems that involve systems of linear equations using a variety of methods.



Applications of Linear Systems

Many problems can be modelled by two or more linear relations, forming a linear system. Any situation that involves a linear system can be solved using three techniques (graphing, substitution, elimination).

The four most most common applications involve:

- numeric or value-based problems
- mixtures and percentages
- times to complete one or more tasks, or
- problems relating speed, distance and time.

Numeric Applications

Example 1: The sum of two numbers have a sum of 26. When the smaller number is tripled, it is two greater than the larger number. Determine the values of the two numbers.

Numeric Applications

Example 1:

Value Applications

Example 2: A handful of quarters and dimes contains 21 coins, for a total of $3.30. Determine the amount of each coin.

Value Applications

Example 2:

Value Applications

Example 3: A retailer orders 40 T-shirts and 21 pairs of jeans from a supplier for $289.00. The next week, he orders 28 T-shirts and 17 pairs of jeans for $223.00. How much does the supplier charge for each item?

Applications Involving Percentages

Example 4: Two sums of money, totalling \$1600, are invested in separate accounts for a year. One account paid 4%/a interest, while the other paid 4%/a. A total of \$94 in interest was earned. How much was invested in each account?

Applications Involving Percentages

Example 4:

Applications Involving Percentages

Example 5: Alex deposits $500 into one account and $1200 into another. At the end of the year, his two accounts total $1754. Gina invests $800 and $300 into the same two accounts, and earning the same interest after one year. What are the annual interest rates for each account?

Applications Involving Percentages

Example 5:

Mixture Problems

Example 5: A mixture of nuts is made by combining cashews ($8.50/kg) with almonds ($3.25/kg). The resulting mixture has a value of $4.93/kg. If 50kg of the mixture is made, what are the masses of the cashews and almonds?

Mixture Problems

Example 5:

Mixture Problems

<u>Example 6:</u> A chemist needs 200 mL of a 1.5% acidic solution. Two other acidic solutions - 0.8% and 3.2% - are available for mixing. How much of each solution should be mixed to produce the desired solution?

Speed, Distance, and Time Problems

Recall the Speed, Distance and Time formula: s = —

Example 7: Two trains, 390 km apart, drive toward each other on parallel tracks. The first train has an average speed of 75 km/h, and the second a speed of 55 km/h. After how long will the trains meet each other? How far from each station will they be?

Speed, Distance, and Time Problems

Example 7:

Speed, Distance, and Time Problems

<u>Example 8:</u> A current flows through a river at a constant rate. A canoeist paddles 6 km upstream (against the current) in 3 hours. The return trip downstream (with the current) takes 2 hours. Determine the speed of the canoeist in still water, and the speed of the current.

Speed, Distance, and Time Problems

Example 8:

Speed, Distance, and Time Problems

<u>Example 9:</u> A student lives 11 km from school. On her morning commute, she walks to a bus stop at an average speed of 6 km/h. She then rides the bus for the remainder of the distance, at an average speed of 45 km/h. If the entire trip takes 32 minutes, what are the times spent walking and riding the bus?

Speed, Distance, and Time Problems

Example 9: