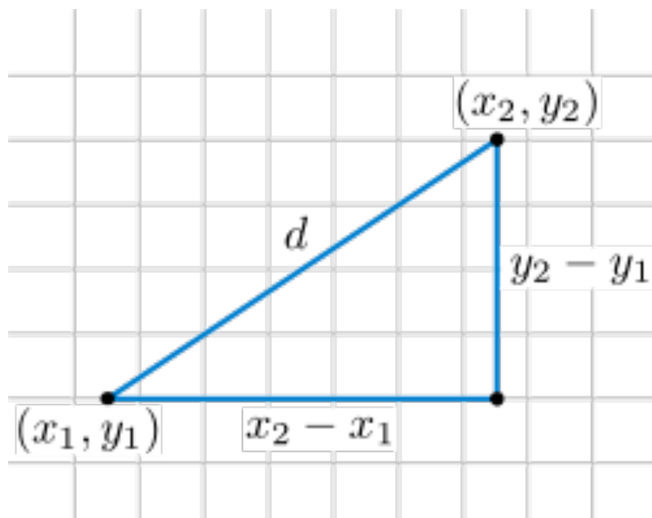
Unit 2: Analytic Geometry

V. Nguyen



Simplifying Radicals

Analytic Geometry 1

* Goal: Simplify radicals (or roots).

Distributive Law of Radicals

For any real numbers a and b, $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Radicals

A **radical**, also called a ***root,*** is typically represented using the form $\sqrt[n]{x}$, where x is the *radicand*, while n is the *index*.

For example, the third root (cube root) of 8 is written $\sqrt[3]{8}$ meaning "the value which, when multiplied by itself three times, gives eight."

If no index is given, the *square root* $\sqrt[2]{x}$ or $\sqrt{x}$ is implied.

Mixed Radicals is the product of a whole number and a radical. For example, $2\sqrt{3} = 2 \times \sqrt{3}$.

Also, $\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$. **Verify** with a <u>calculator</u>.

Simplifying Radicals

<u>Examples:</u> *Simplify. <u>Tips:</u> 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, …

a) $\sqrt{40}$

b) $\sqrt{15}$

c) $\sqrt{128}$

d) $\sqrt{252}$

e) $-\sqrt{56}$

f) $\sqrt{-144}$

Length of a Line Segment

Investigation:

Plot the points $A(2,3)$ and $B(10,7)$, and draw line segment AB. Find the third point C such that line segments AC and BC make a right triangle with AB as the hypotenuse.

a) How long are AC and BC?

b) How long is AB?

Length of a Line Segment

Length of a HORIZONTAL Line Segment:

The length of the **horizontal** line segment PQ connecting $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by $|PQ| = x_2 - x_1$

Length of a VERTICAL Line Segment:

The length of the **vertical** line segment PQ connecting $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by $|PQ| = y_2 - y_1$

Length of a Line Segment

Example 1: Determine $|JK|$ for $J(5,-2)$ and $K(11,-2)$.

Example 2: Determine the value of k if $|EF| = 12$ for $E(-3, 1)$ and $F(-3, k)$.

Length of a Line Segment

Length of a Line Segment:

The length of the **line segment** PQ connecting $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Sometimes, the length of a line segment is phrased "the distance between two points", and the formula above is best known as the **Distance Formula**.

Example 1: **Verify** that $|AB| = 4\sqrt{5}$ for $A(2,3)$ and $B(10,7)$.

Length of a Line Segment

__Example 2:__ **Determine** the distance between the points $G(1,-4)$ and $H(-6,-3)$.

Length of a Line Segment

Example 3:

Middlefield is 12 km South and 5 km West from the Markham Museum. Markville is 7 km South and 11 km East of the Markham Museum. If a helicopter flies directly from Middlefield to the Markham Museum, how far does it fly?

Midpoint of a Line Segment

Investigation: Draw a right triangle ABC given that $A(2,3)$, $B(10,7)$, and $C(10,3)$. Determine the midpoints on each line segment that are *equidistant* from the *endpoints*) of AC, BC, and AB. How are the coordinates related?

Midpoint of a Line Segment

Midpoint of a Horizontal Line Segment

If PQ is a horizontal line segment with endpoints at $P(x_1,k)$ and $Q(x_2,k)$, the midpoint M is located at $M($ $\qquad\qquad$ $, k)$.

Midpoint of a Vertical Line Segment

If PQ is a vertical line segment with endpoints at $P(k,y_1)$ and $Q(k,y_2)$, the midpoint M is located at $M(k,$ $\qquad\qquad$ $)$.

Example: Determine the coordinates of the midpoint of $A(3,7)$ and $B(3,19)$.

Midpoint of a Line Segment

Midpoint of a Line Segment

If *PQ* is a line segment from $P(x_1,y_1)$ and $Q(x_2,y_2)$, then the midpoint of *PQ* is located at *M*(,).

Example 1: Determine the coordinates of the midpoint of the line segment connecting *A*(4,9) and *B*(14,3).

Example 2: Determine the coordinates of the midpoint of the line segment connecting *J*(-15,13) and *K*(-3,-7).

Midpoint of a Line Segment

Example 3: If $M(5,-2)$ is the midpoint of PQ and P is at (-8,11), determine the coordinates of Q.

Example 3: Determine if the line $y = -3x + 5$ bisects the line segment AB, given $A(-4,7)$ and $B(8,-9)$.

Midpoint of a Line Segment

Example 3: If $M(5,-2)$ is the midpoint of PQ and P is at (-8,11), determine the coordinates of Q.

Midpoint of a Line Segment

<u>Example 4:</u> Determine if the line $y = -3x + 5$ bisects the line segment AB, given $A(-4,7)$ and $B(8,-9)$.

Classifying Triangles

Triangles are generally classified in two ways:

1. Magnitude of its <u>sides:</u>

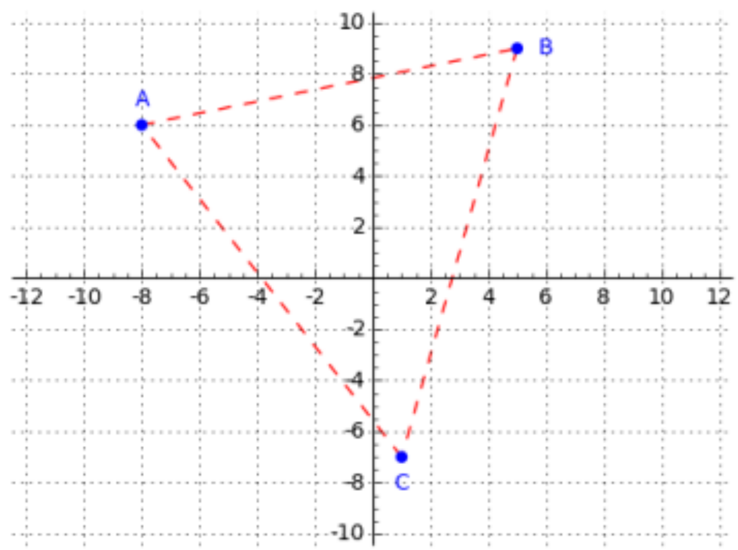
- Equilateral (three equal sides/angles)
- Isosceles (two equal sides)
- Scalene (all side lengths are distinct)

2. Magnitude of its <u>angles:</u>

- Right (one 90° angle)
- Acute (all angles less than 90°)
- Obtuse (one angle greater than 90°)

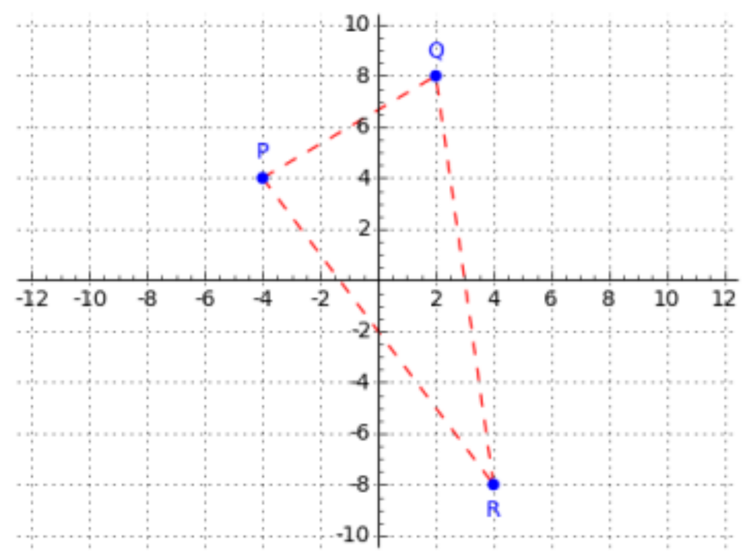
Classifying Triangles

Example 1: Classify the triangle with vertices A(-8,6), B(5,9) and C(1,-7) as equilateral, isosceles or scalene.



Classifying Triangles

<u>Example 2:</u> Determine whether the triangle with vertices *P*(-4,4), *Q*(2,-8) and *R*(4,-8) contains a right angle.



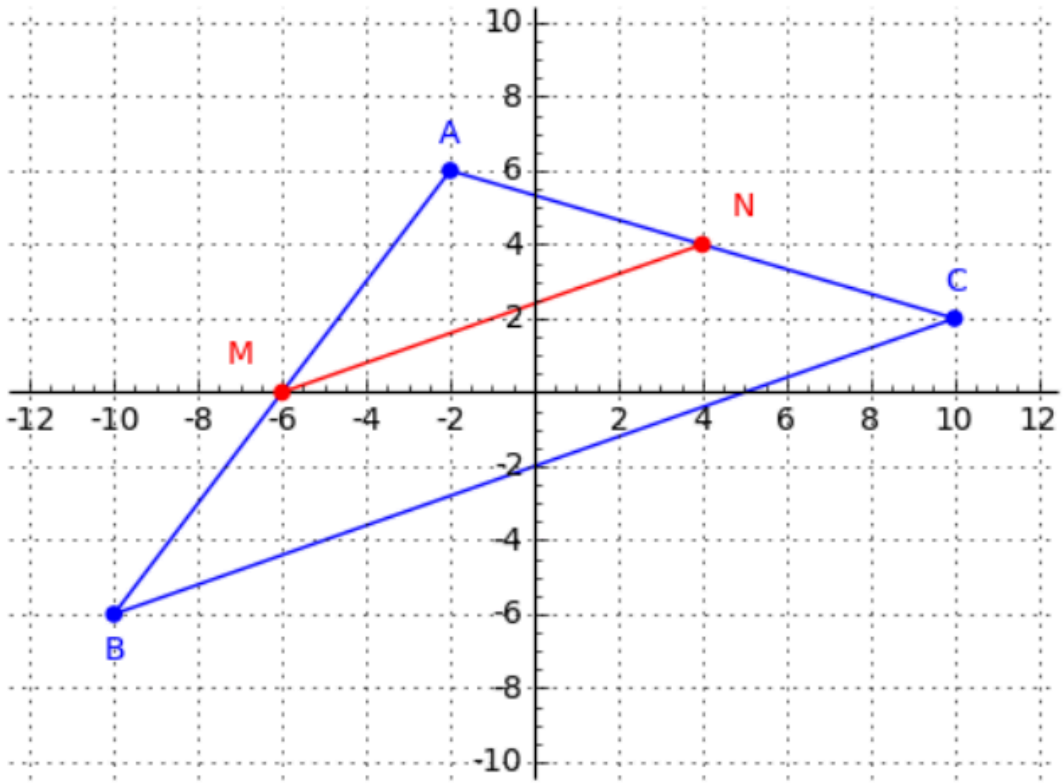
Verifying Triangles

Example 3: Show that it is not possible for the points $E(-5,8)$, $F(-2,6)$ and $G(4,2)$ to form a triangle.

Properties of Triangles

<u>Triangle Midpoint Theorem:</u>

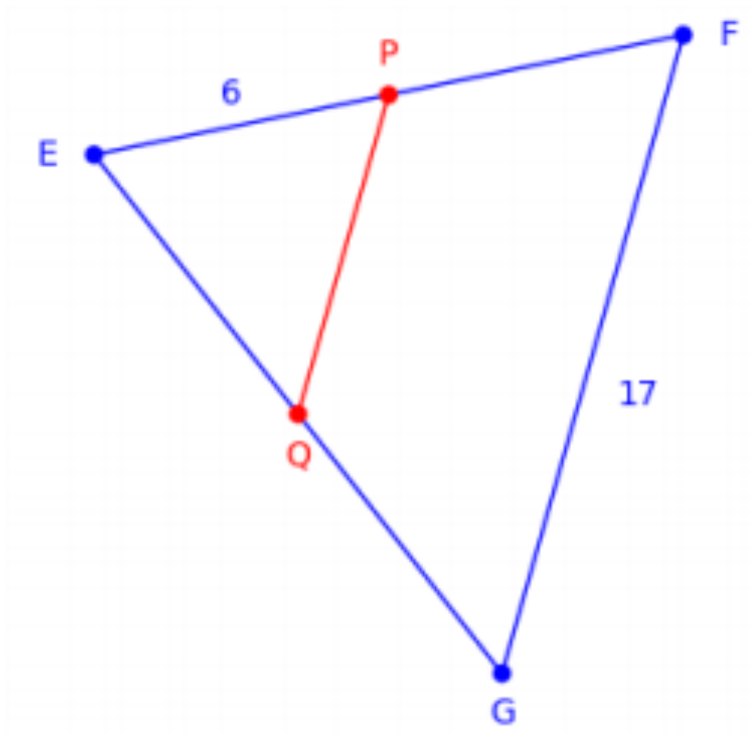
If M and N are the midpoints of AB and AC in $\triangle ABC$, then $|BC| = 2|MN|$ and BC || MN . Note (|| - *parallel to*)



Properties of Triangles

Example:

In the diagram below, P and Q are the midpoints of EF and EG. Determine $|EF|$ and $|PQ|$.



Classifying Quadrilaterals

We often classify *quadrilaterals* using slopes, midpoints or lengths. A quadrilateral is any *four-sided* polygon. They can be *convex* (no angle is greater than 180°). Special types of quadrilaterals have unique properties.

1. **Parallelogram:** two pairs of parallel sides, opposite lengths are equal.

2. **Rectangle:** parallelogram that contains four 90° angles.

Classifying Quadrilaterals

3. **Trapezoid:** exactly one pair of parallel sides. If the two non-parallel sides are equal in length, it is an *isosceles trapezoid*. Otherwise it is a *scalene trapezoid*.

4. **Kite:** when all interior angles are less than 180°, it is a *kite*. When one angle is greater than 180°, it is a *chevron*.

Classifying Quadrilaterals

Example 1: Verify that the quadrilateral $ABCD$ with vertices at A(-1,4), B(6,1), C(3,-6) and D(-4,-3) is a square.

Classifying Quadrilaterals

Example 2: Verify that the quadrilateral *EFGH* with vertices at *E*(-8,2), *F*(4,6), *G*(6,-2) and *H*(-6,-6) is a parallelogram, but not a rhombus or a rectangle.

Classifying Quadrilaterals

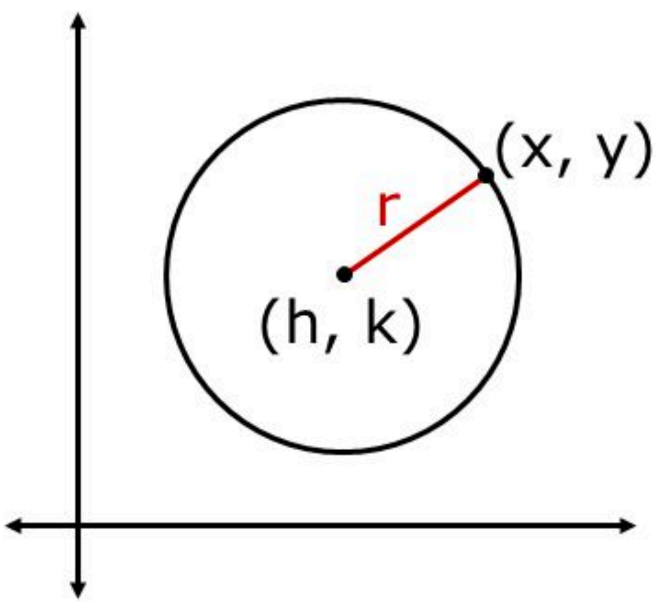
Example 3: Classify the quadrilateral *PQRS* with vertices at *P*(-6,2), *Q*(6,6), *R*(2,-6) and *S*(-8,-8).

Classifying Quadrilaterals

Example 4: A quadrilateral has three vertices at $A(-2,2)$, $B(4,0)$, and $C(6,-4)$. Determine the coordinates of D so that the quadrilateral is a parallelogram.

Unit 2: Analytic Geometry 2

V. Nguyen

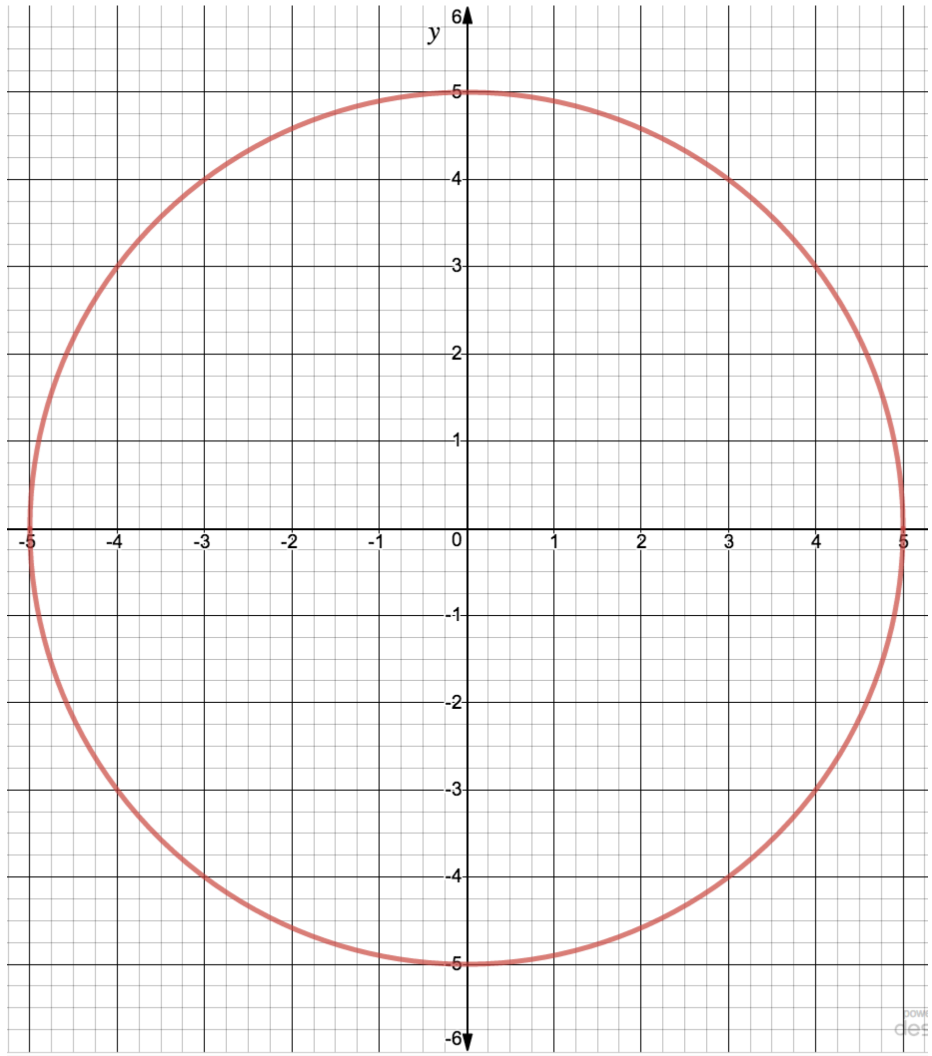


Use the Distance Formula to write this.

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

Equations of Circles

Consider a circle centred at the origin with a radius of 5 units.



Equations of Circles

In all cases, the hypotenuse of any right triangle formed is a radius of the circle, r.

If the horizontal arm of a right triangle formed has a length of x units, and the vertical arm has a length of y units, then the length of the hypotenuse can be calculated using the Pythagorean Theorem.

This gives us an equation for a circle, centred at the origin.

$$x^2 + y^2 = r^2$$

Equations of Circles

Example 1: Determine the equation of a circle, centred at the origin, with a radius of 3 units.

Example 2: A circle has equation $x^2 + y^2 = r^2$. Determine the length of its radius.

Equations of Circles

Example 3: Determine the equation, and length of the radius, of a circle centred at the origin that passes through $P(-2,3)$.

*Graph the circle and point to confirm calculations.

Equations of Circles

Example 4: Determine whether $P(6,-8)$ is on, inside, or outside of the circle with radius 10.

*Graph the circle and point to confirm calculations.

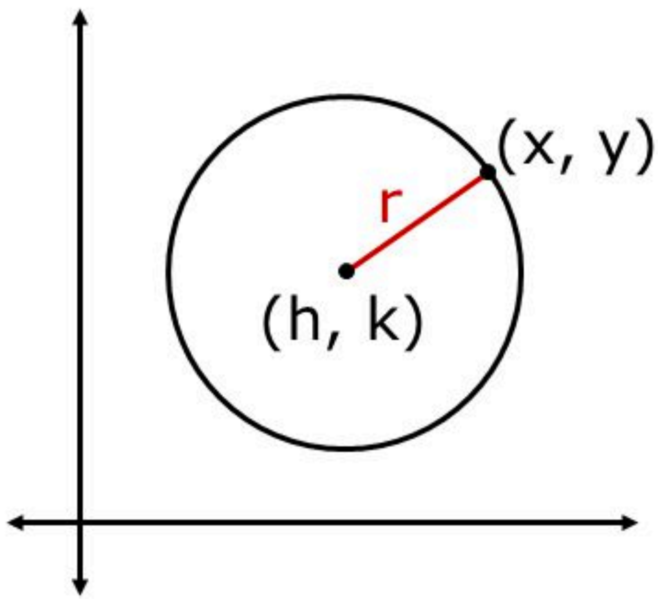
Equations of Circles

Example 5: Determine whether $P(-4,5)$ is on, inside, or outside of the circle with equation $x^2 + y^2 = 36$

*Graph the circle and point to confirm calculations.

Equations of Circles

Summary:



Use the Distance Formula to write this.

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

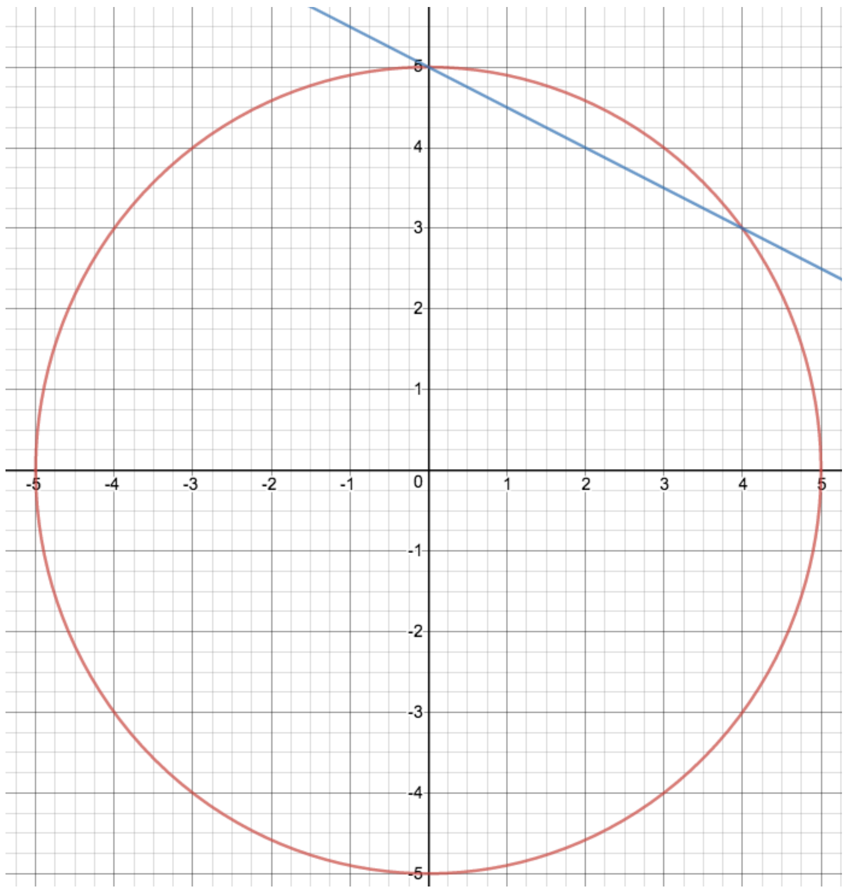
Tangents

Minds On:

Determine the equation and length of a radius of a circle, centred at the origin, that passes through the point $P(9,-3)$.

Secants and Chords

Consider the circle and line as shown.

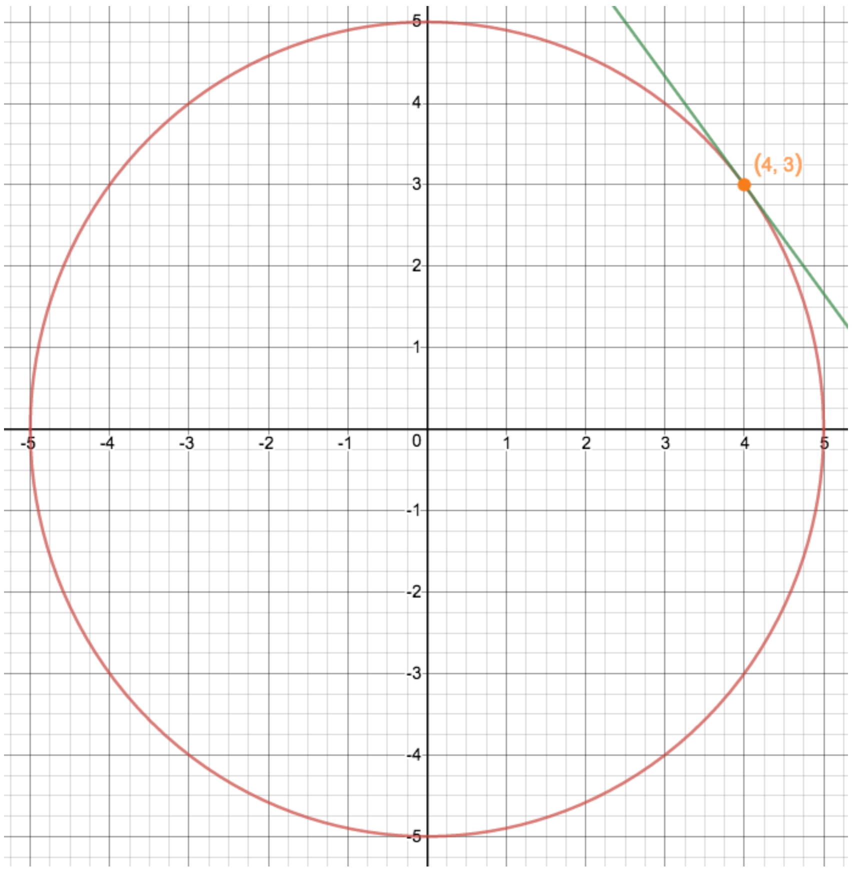


Observations:

Equation:

Tangents

Now consider the circle and line as shown.



Observations:

Equation: ?

Tangents

Equation:

Tangents

Equation of a Tangent to a Circle Centre at the Origin:

$$y = -\frac{x_p}{y_p}x + \frac{\left(x_p^2 + y_p^2\right)}{y_p}$$

Using the previous example, the equation of the tangent at (4,3) is:

$y =$

Tangents

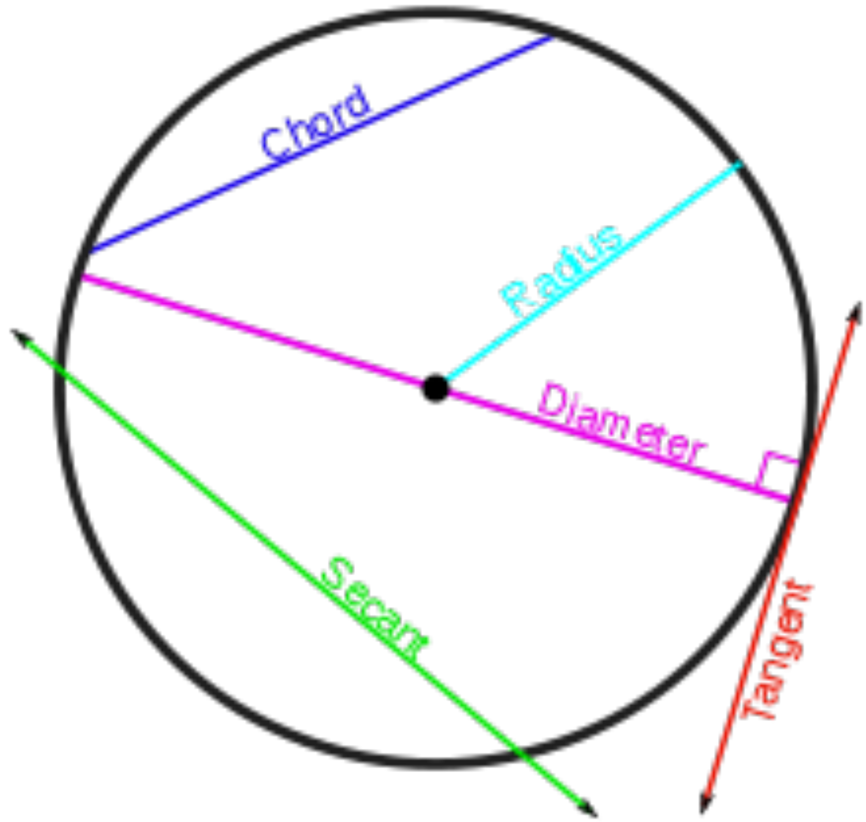
__Example 1:__ The line $y = 3x - 10$ is tangent to a circle, centred at the origin, when $x = 3$. Determine the equation and radius of the circle.

Tangents

Example 2: Determine the equation of the tangent to the circle, centred at the origin with radius 13 units, when $x = 5$.

Tangents

Summary:

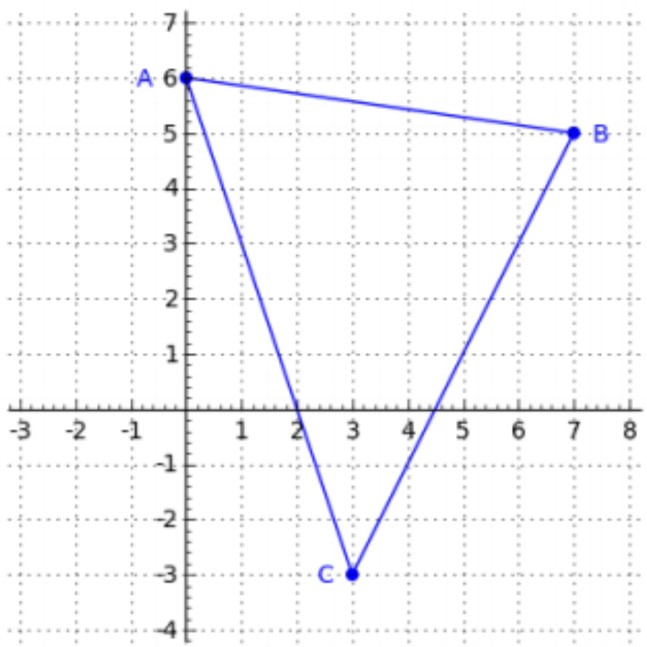


Circumcentre of a Triangle

Recap: Determine the equation of the right bisector of the line segment from $A(-4,-7)$ to $B(10,1)$.

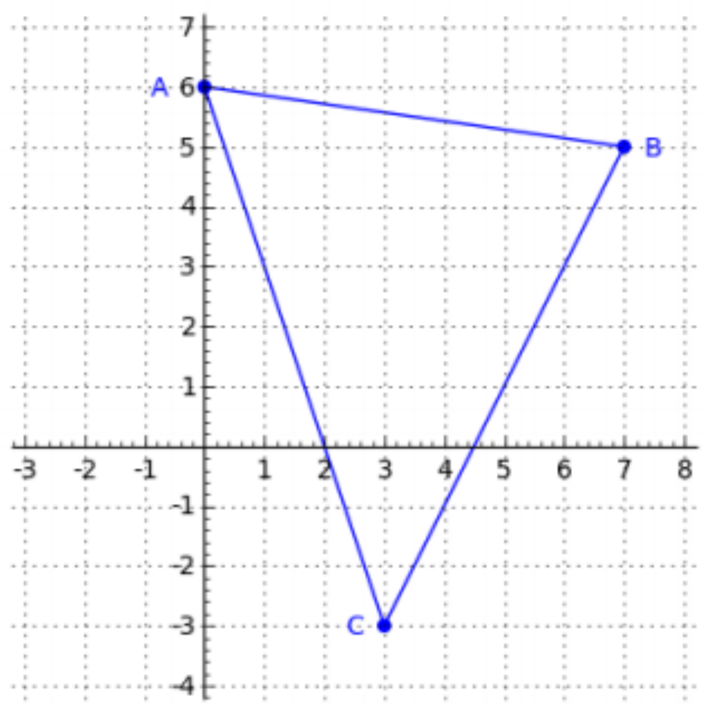
Right Bisectors In a Triangle

Consider the triangle below with vertices at $A(0,6)$, $B(7,5)$, and $C(3,-3)$. Let's construct the right bisectors of AC and BC.



Right Bisectors In a Triangle

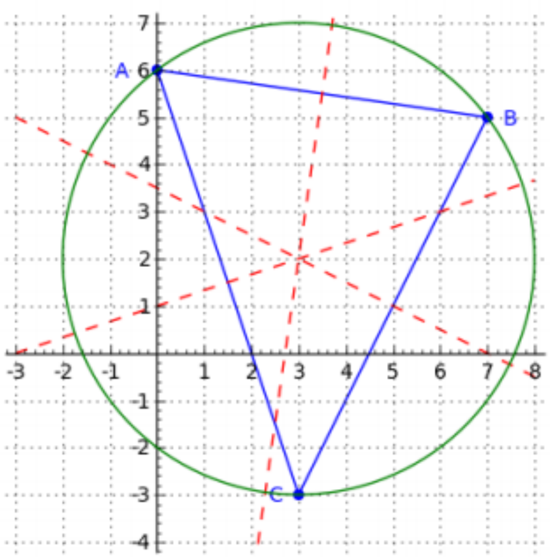
Since the right bisectors have *different* slopes, they must intersect somewhere. Use substitution to determine the POI.



Note: the distance from the circumcentre, *P*, to each vertex should be the same. *HW:* Verify that it is 5 for this example.

Circumcentre of a Triangle

The right bisectors of the sides of a triangle intersect at a point called the circumcentre. The circumcentre is equidistant from all three vertices of the triangle.



Steps to find Circumcentre:

1. Determine midpoint of one side.
2. Determine slope of that side.
3. Determine perpendicular slope to that side.
4. Use perpendicular slope and midpoint to determine equation of the right bisector of that side.
5. Repeat steps 1-4 for another side.
6. Find the point of intersection of the two right bisectors.

Circumcentre of a Triangle

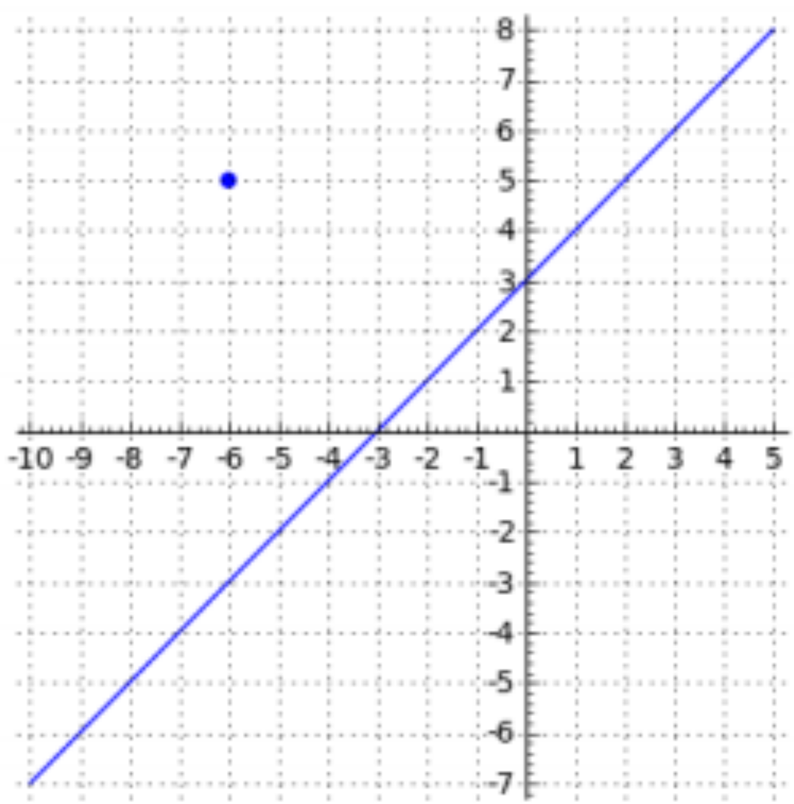
Example: Determine the radius of the circle that passes through $P(-7,2)$, $Q(11,-10)$ and $R(11,14)$.

Shortest Distance

Recap: Determine the equation of the altitude from $A(-4,7)$, given that $B(-8,0)$ and $C(2,-2)$.

Shortest Distance

Investigation 1: Determine the shortest distance from $P(-6,5)$ to the line $y = x + 3$.



Shortest Distance

<u>Investigation 2:</u> Determine the area of the triangle with vertices $A(3,6)$, $B(-7,0)$ and $C(5,-3)$.