MPM2D: Principles of Mathematics

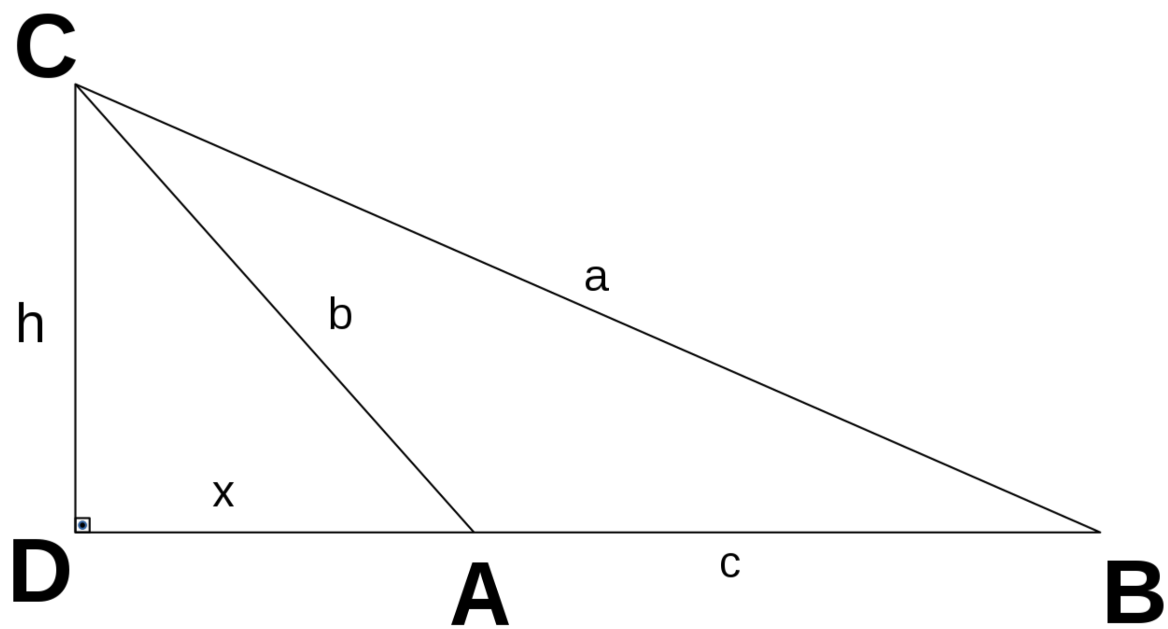
# Unit 4: Trigonometry

V. Nguyen

# Similarity & Proportionality

## Trigonometry

- Goal: Understand that a *ratio* is a comparison between two quantities.



# Solving Proportions

A **proportion** is a statement that equates two or more ratios.

For example, ________________ is a proportion since the ratio can be reduced.

In general, two ratios are equal if there is some *scale factor* ***k***

such that $\frac{k}{k} \cdot \frac{a}{b} = \frac{c}{d}$

In this case, since __________________, $k$ = ___.

Many proportions can be solved by finding the scale factor *k*, then we can find any missing values.

# Solving Proportions

**Examples:** *Solve

a) $\frac{x}{15} = \frac{2}{5}$

b) $\frac{x}{12} = \frac{5}{8}$

c) $\frac{x}{4} = \frac{5}{7}$

d) $\frac{x+1}{8} = \frac{4}{3}$

e) $\frac{-x-1}{5} = \frac{2}{5}$

# Word Problems Involving Proportions

Many types of word problems involve proportions. Information will usually be given about one ratio, while the remaining ratio will require solving.

Proportion problems usually have the format "**if *a* is to *b*, then *c* is to …**".

Remember, in math… there can be more than one way to solve problems. As long as the ratios are valid, solving a proportion using the previous techniques should work.

# Word Problems Involving Proportions

**Example 1:** If it costs \$48.00 to buy 5 shirts, how much should it cost to buy a dozen shirts?

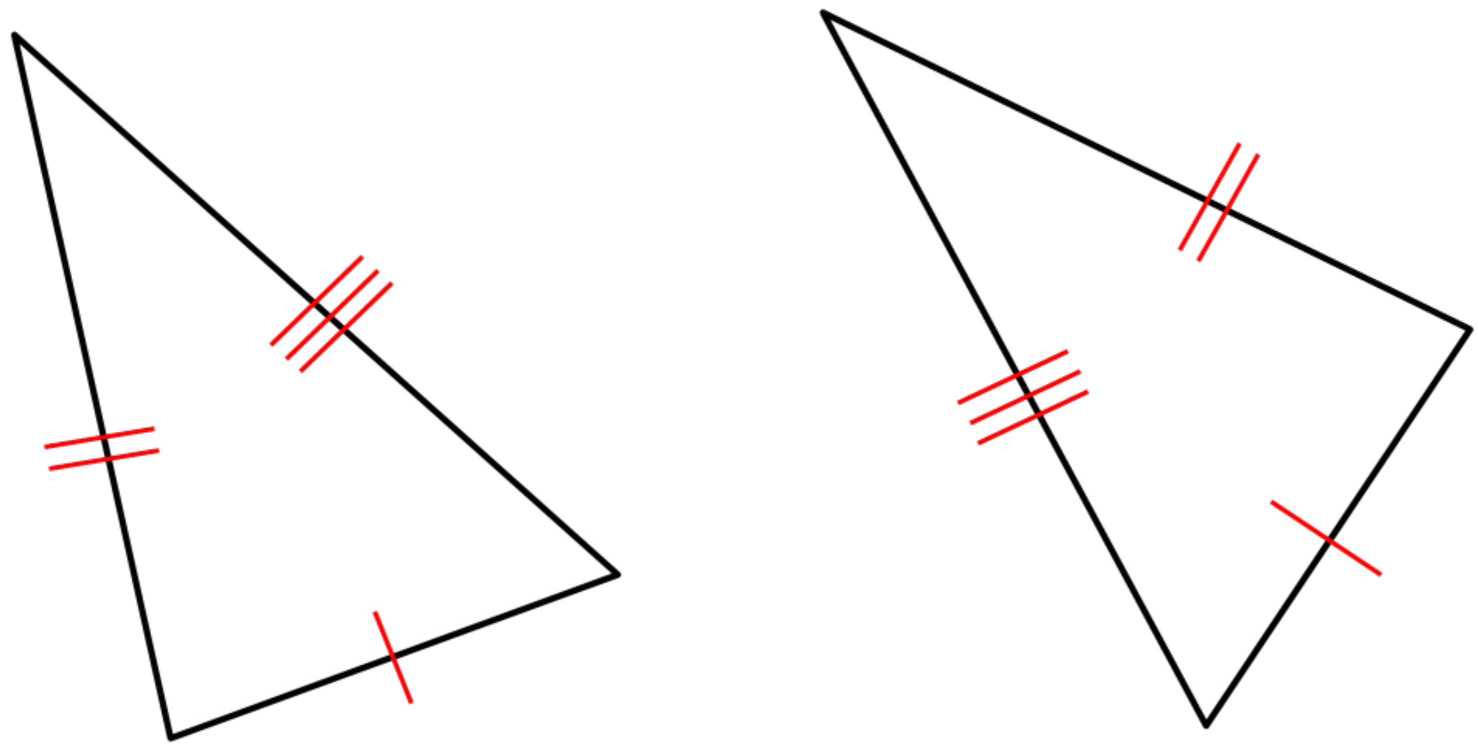
**Example 2:** A pallet of bricks has a mass of 10 000 kg. After 1680 bricks have been used, the pallet has a mass of 5800. How many bricks were on the pallet?

# Congruent & Similar Triangles

* Goal: Understand the three conditions that will result in *congruent* triangles **and** three conditions that will result in *similar* triangles.

# Congruent Triangles

2 triangles are *congruent* if they have the same <u>shape</u> & <u>size</u>. To meet both of these criteria, congruent triangles have equal corresponding angles and equal corresponding sides.



# Congruent Triangles

**3 distinct conditions** that will result in **congruent triangles**.

**1. Side-Side-Side Congruency (SSS)**

If $|AB| = |DE|$, $|AC| = |DF|$ and $|BC| = |EF|$, then $\triangle ABC \approx \triangle DEF$.

**2. Side-Angle-Side Congruency (SAS)**

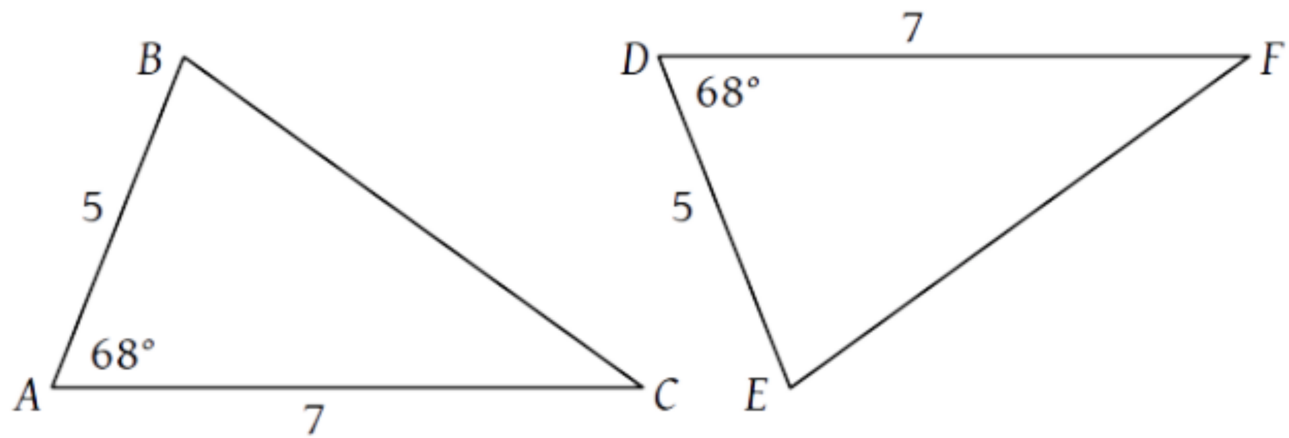
If $|AB| = |DE|$, $|AC| = |DF|$ and $\angle A = \angle D$, then $\triangle ABC \approx \triangle DEF$.

**3. Angle-Side-Angle Congruency (ASA)**

If $\angle A = \angle D, \angle B = \angle E$, and $|AB| = |DE|$, then $\triangle ABC \approx \triangle DEF$.

# Congruent Triangles

**Example 1:** State why[2] $\triangle ABC \approx \triangle DEF$.

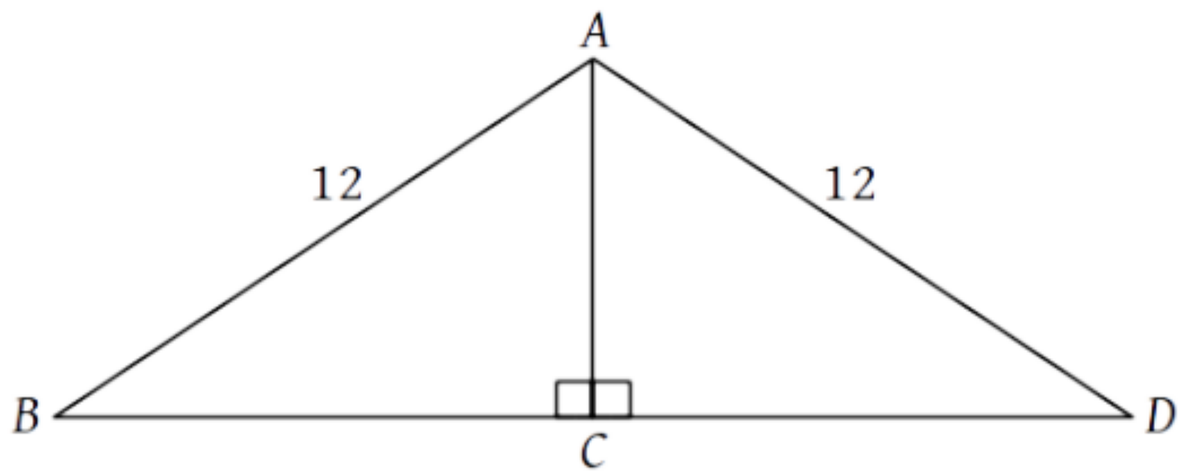



**[2]Side-Angle-Side Congruency (SSS)**

If $|AB| = |DE|$, $|AC| = |DF|$ and $|BC| = |EF|$, then $\triangle ABC \approx \triangle DEF$.

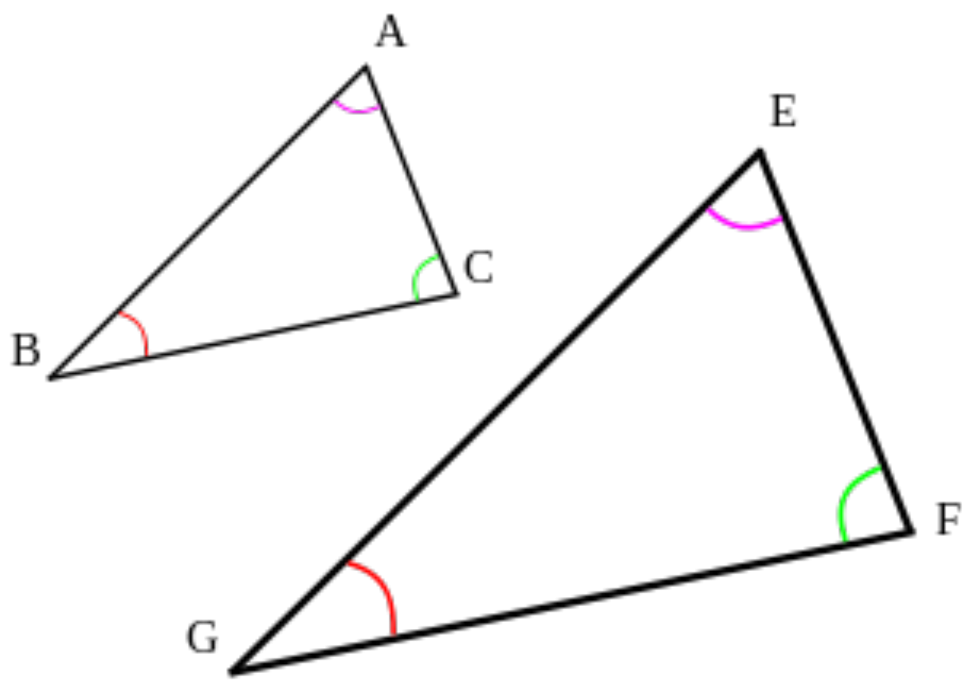
# Congruent Triangles

**Example 2:** State why $\triangle ABC \approx \triangle ADC$.



# Similar Triangles

2 triangles are *similar* if they have the same overall shape.
Triangles with equal corresponding angles will have the same shape, but may be different sizes.



# Similar Triangles

**3 distinct conditions** that will result in **similar triangles**.

## 1. Side-Angle-Side Similarity (SAS~)

If $|AB| = k|DE|$, $|AC| = k|DF|$ and $\angle A = \angle D$ then $\triangle ABC \sim \triangle DEF$.

## 2. Angle-Angle Similarity (AA~)

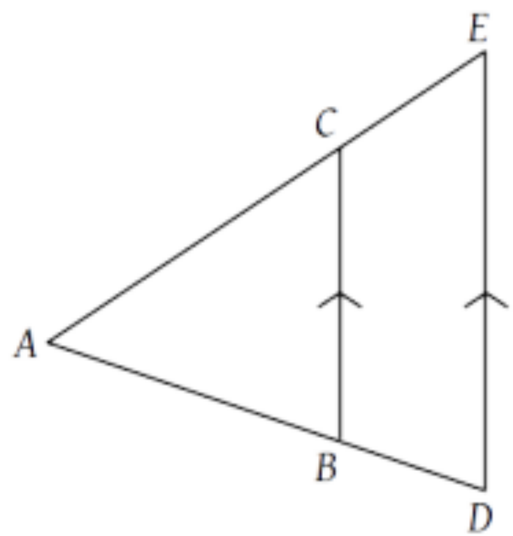
If $\angle A = \angle D, \angle B = \angle E,$ then $\triangle ABC \sim \triangle DEF$.

## 3. Side-Side-Side Similarity (SSS~)

If $|AB| = k|DE|$, $|AC| = k|DF|$ and $|BC| = k|\mathrm{EF}|$,
then $\triangle ABC \sim \triangle DEF$.

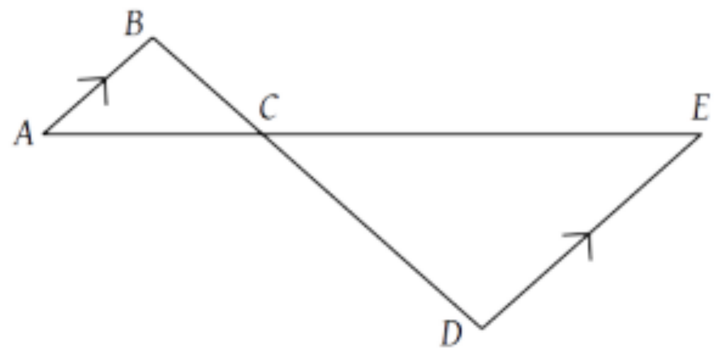
# Similar Triangles

**<u>Example 1:</u>** State why $\triangle ABC \sim \triangle ADE$.

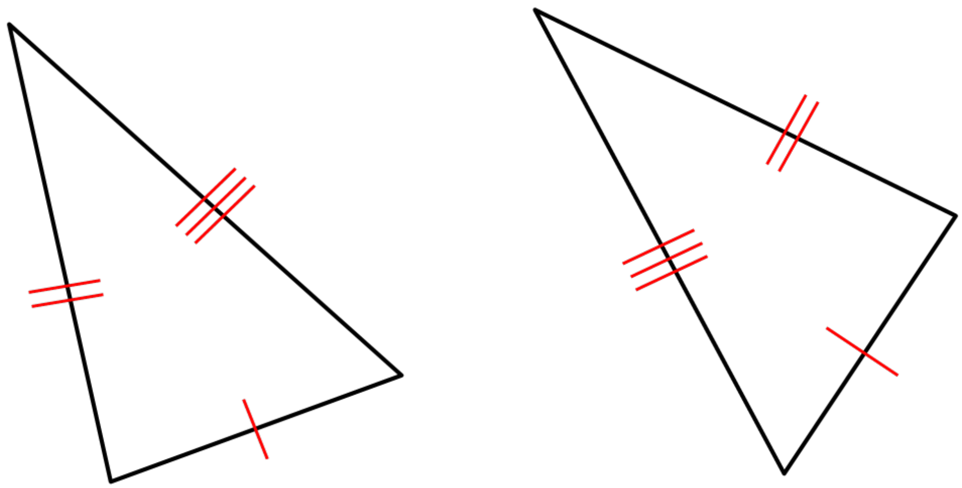


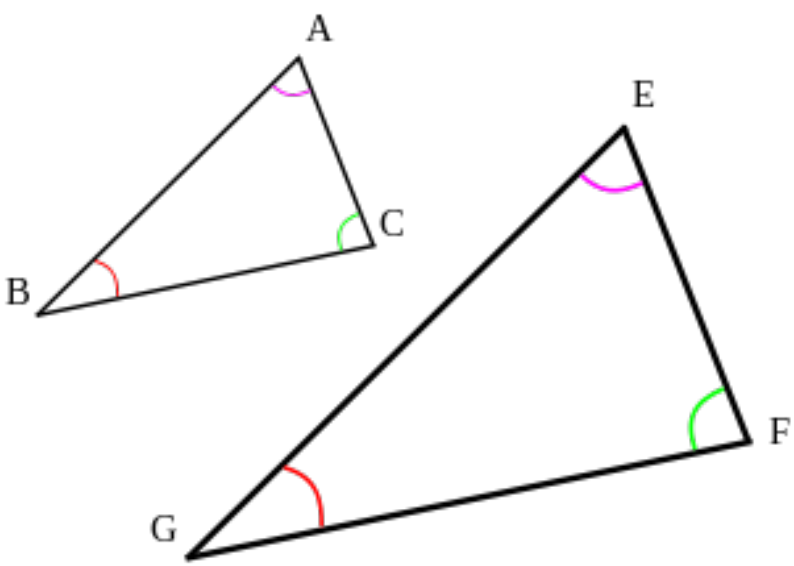
# Similar Triangles

**Example 2:** State why $\triangle ABC \sim \triangle EDC$.



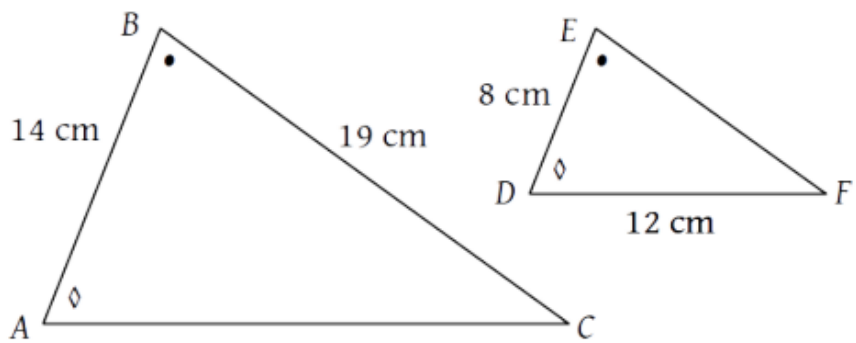
# Congruent & Similar Triangles Summary





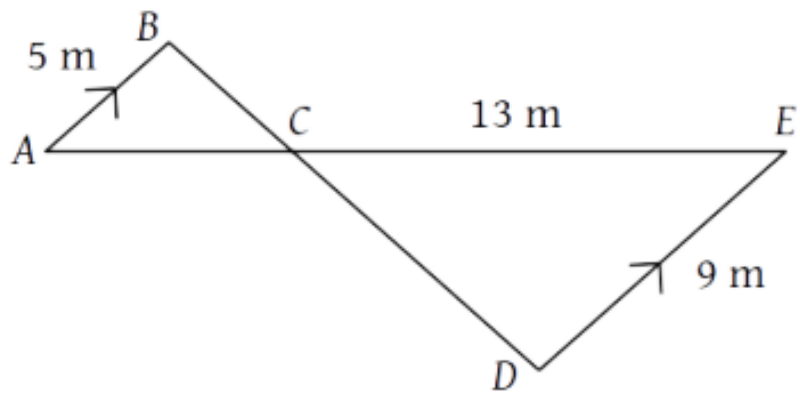
# Solving Similar Triangles

**<u>Example 1:</u>** $\triangle ABC \sim \triangle DEF$. Determine $|AC|$ and $|EF|$.



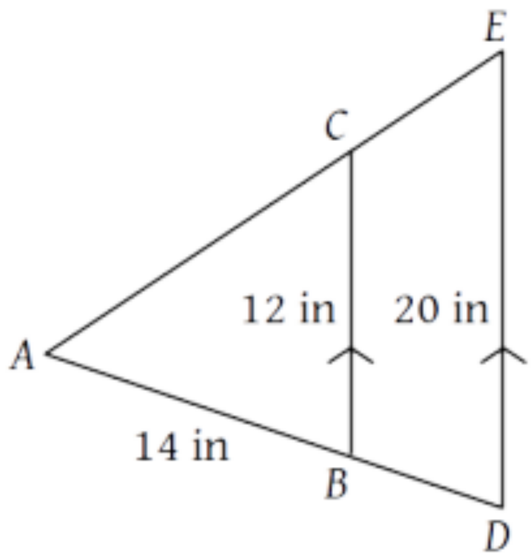
# Solving Similar Triangles

**<u>Example 2:</u>** Determine $|AE|$.



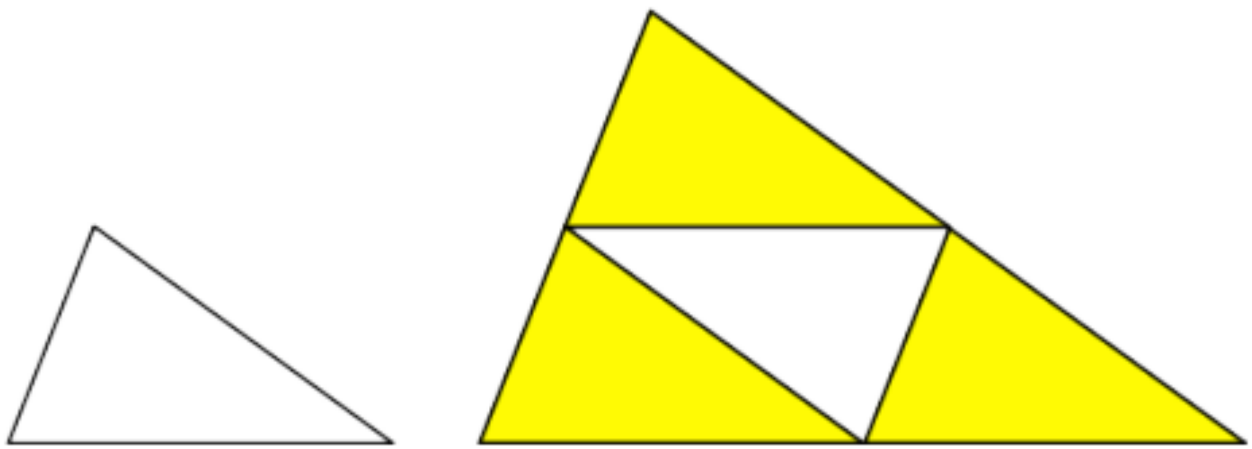
# Solving Similar Triangles

**Example 3:** Determine $|BD|$.



# Areas of Similar Triangles

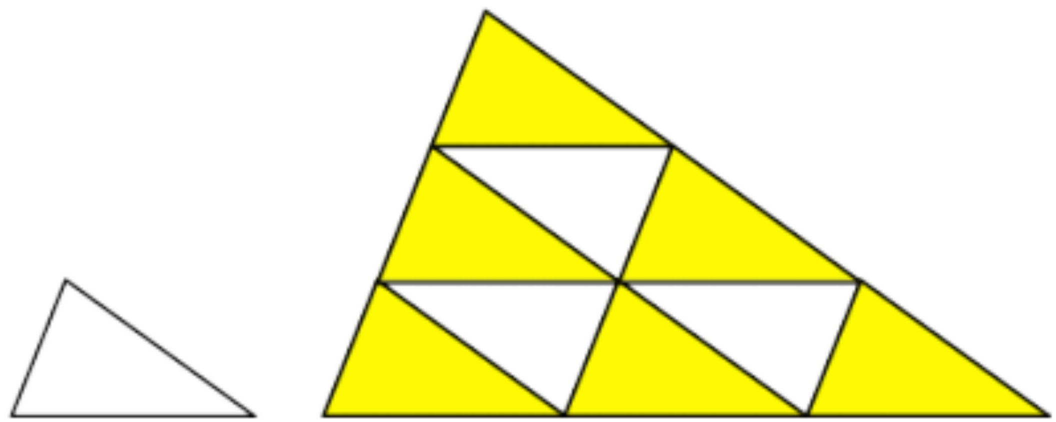
What happens to the area of a triangle when its dimensions are doubled?



When the dimensions are doubled, the area is ___________.

# Areas of Similar Triangles

What happens to the area of a triangle when its dimensions are tripled?



When the dimensions are tripled, the area is increased by a factor of ___.

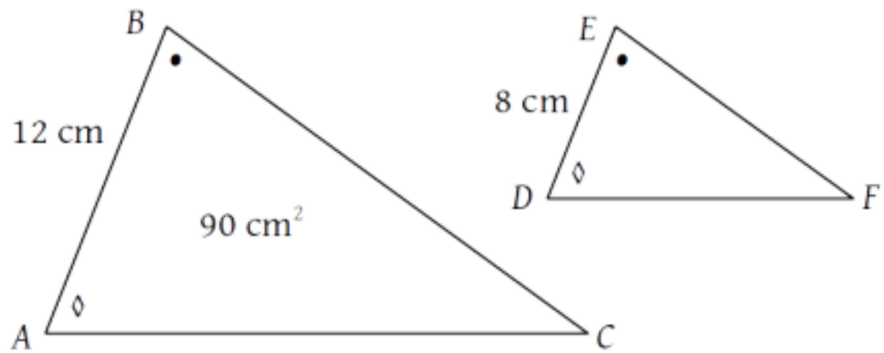
# Areas of Similar Triangles

In general, a triangle whose dimensions are enlarged  factor of $k$ will have an area ____ times greater.

Any ratio of corresponding sides can be used, so choose one that is easiest to work with.

# Areas of Similar Triangles

**Example 3:** Determine the area of triangle DEF.



# Applications of Similar Triangles

Many situations can be modelled using similar triangles, e.g. the heights of inaccessible objects (cliffs, trees, etc.) can be estimated using smaller models.

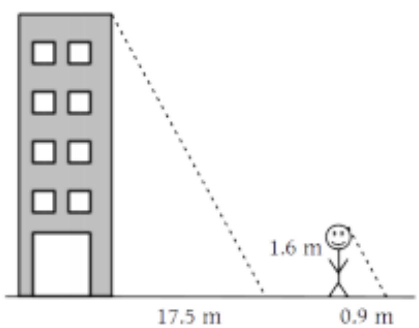
As long as we are able to establish a *proportion* with **three known quantities**, we can solve for a **fourth quantity**.

Of course, this only applies if the triangles are similar.
In some cases, we need to make some assumptions that may not be 100% accurate in order to ensure this.

# Applications of Similar Triangles

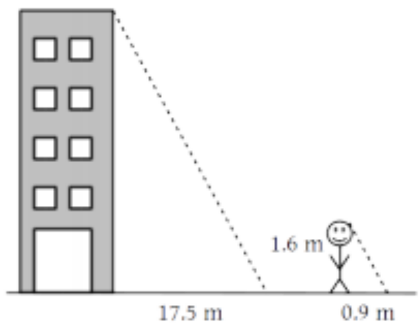
**Example 1:** At 3:00 pm, a building casts a shadow 17.5 m long. At the same time, a 1.6 m student casts a shadow 0.9 m long. Approximately how tall is the building?

Assuming the angle made by the sun is the same for both the building and the student (it isn't... but it's pretty close), we can draw two triangles that are *similar* due to AA~.



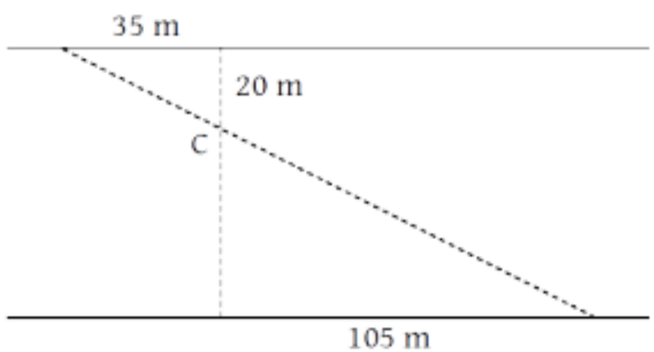
# Applications of Similar Triangles

## Example 1:



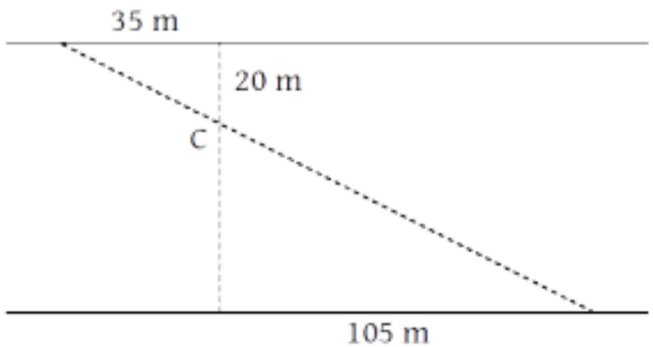
# Applications of Similar Triangles

**Example 2:** A channel marker in a river is located 20 m out from one shore. From a point 35 m down shore, the line of sight to another point 105 m the other way on the opposite shore passes through the marker, as shown. How wide is the river?



# Applications of Similar Triangles

## Example 2:



# Applications of Similar Triangles

**Example 3:** Two corresponding sides in two similar triangles have lengths of 10 cm and 25 cm. If the perimeter of the smaller triangle is 48 cm, what is the perimeter of the larger one?

# Applications of Similar Triangles

## Example 3:

# Applications of Similar Triangles

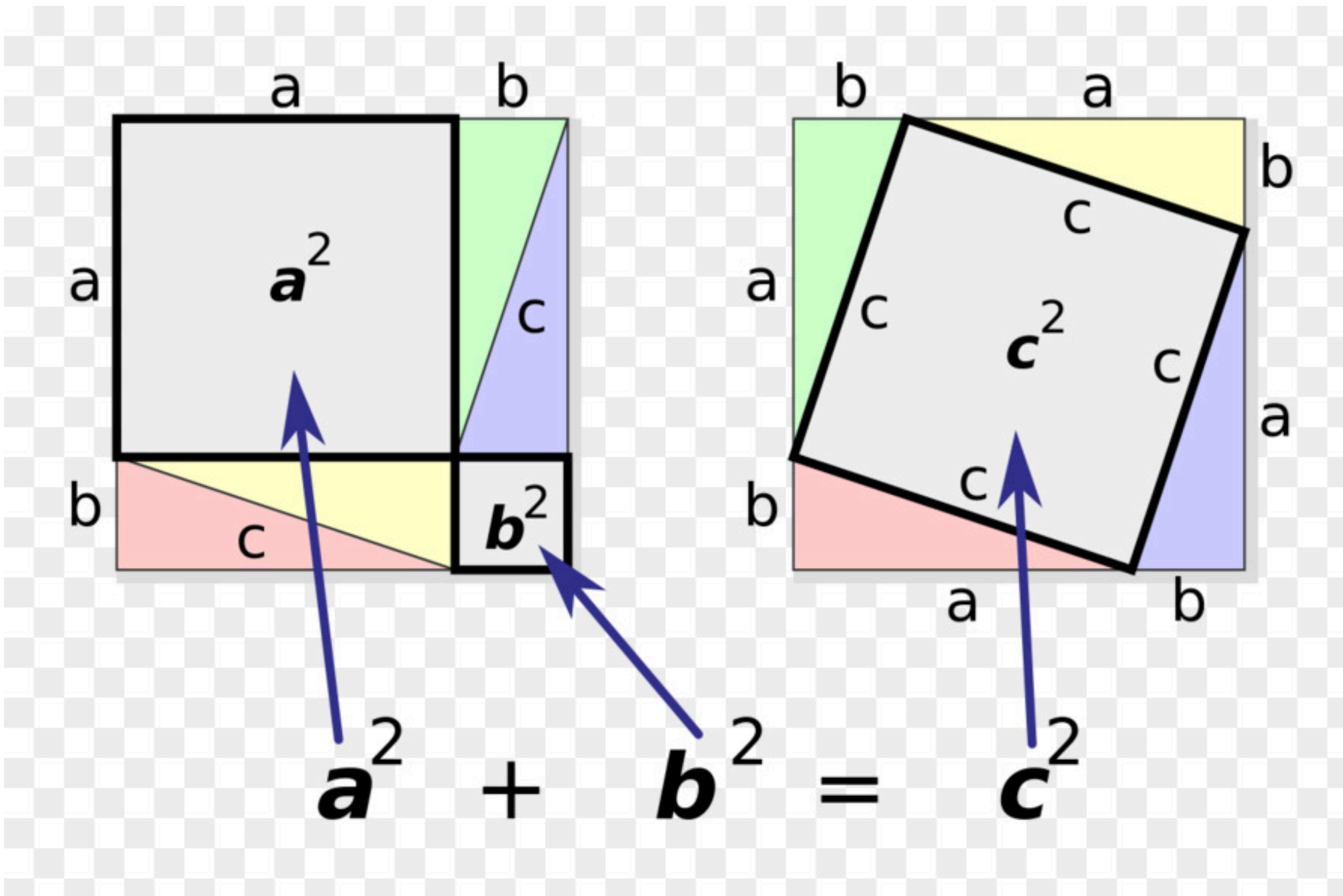
**<u>Example 4:</u>** A triangular garden with side lengths of 4 m, 5 m and 6 m is enlarged so that it holds ten times as many flowers. What are its new dimensions?

# Applications of Similar Triangles

## Example 4:

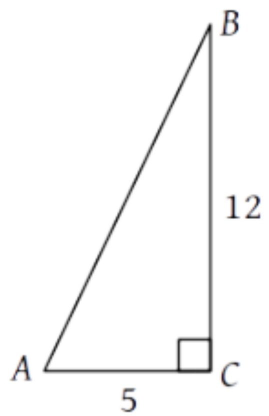
# The Pythagorean Theorem

Consider four congruent triangles with arms *a* and *b* as well as hypotenuses *c*. They can be arranged in many ways, including:

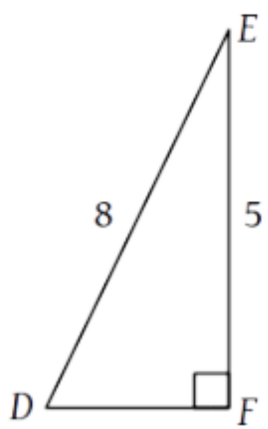


# Pythagorean Theorem

**Example 1:** Determine $|AB|$.

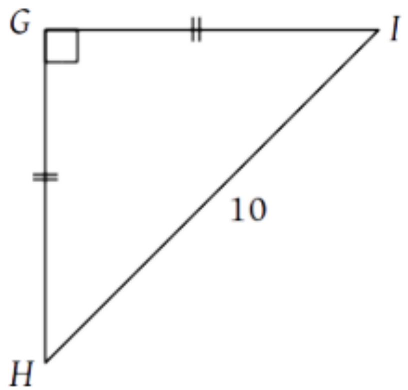



**Example 2:** Determine $|\text{DF}|$.

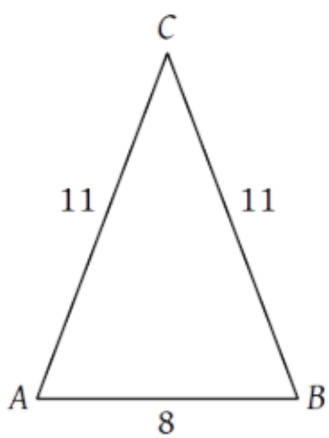


# Pythagorean Theorem

**Example 3:** Determine $|GH|$.

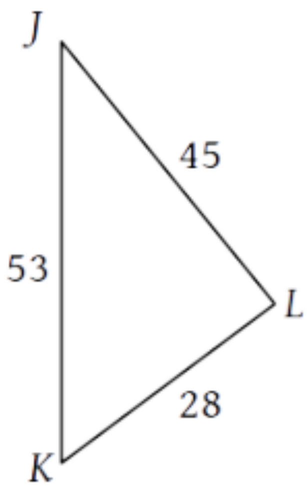



**Example 4:** Determine the area of $\triangle ABC$.



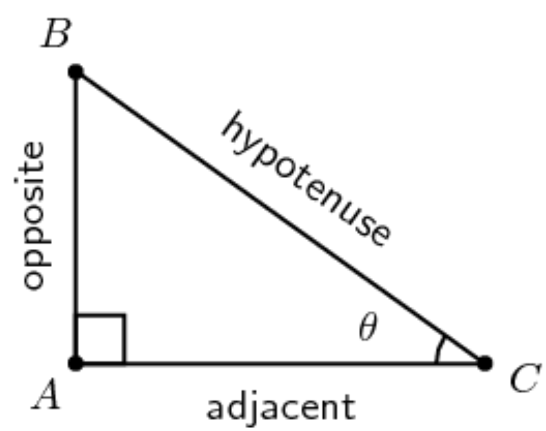
# Pythagorean Theorem

**Example 5:** Verify that $\triangle JKL$ contains a right angle.



# Primary Trigonometric Ratios

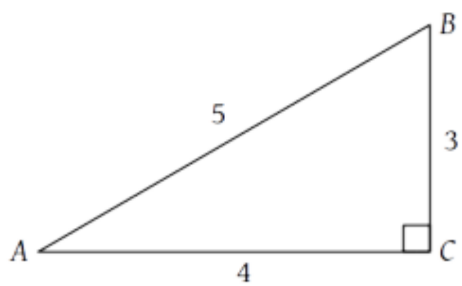
In the right triangle ABC, three sides have been labelled based on their position relative to angle C, in this case denoted by the Greek letter, $\theta$. Remember, the hypotenuse is *always* across from the right angle. There are 6 total, we are focusing on the first 3 for this course.



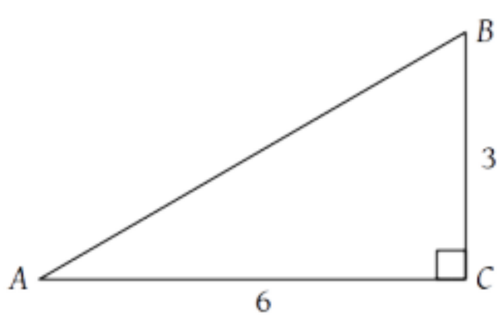

| Name | Abbreviation | Ratio |
|---|---|---|
| Sine | Sin | $\text{Sin}\angle\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$ |
| Cosine | Cos | $\text{Cos}\angle\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$ |
| Tangent | Tan | $\text{Tan}\angle\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$ |
| Secant | Sec | $\text{Sec}\angle\theta = \dfrac{\text{Hypotenuse}}{\text{Adjacent}}$ |
| Cosecant | Csc | $\text{Csc}\angle\theta = \dfrac{\text{Hypotenuse}}{\text{Opposite}}$ |
| Cotangent | Cot | $\text{Cot}\angle\theta = \dfrac{\text{Adjacent}}{\text{Opposite}}$ |

# Primary Trigonometric Ratios

**Example 1:** State the three primary trigonometric ratios for $\angle A$ in $\triangle ABC$.




**Example 2:** State the three primary trigonometric ratios for $\angle A$ in $\triangle ABC$.



# Using Trigonometric Ratios
# Part 1: Solving for Unknown Sides

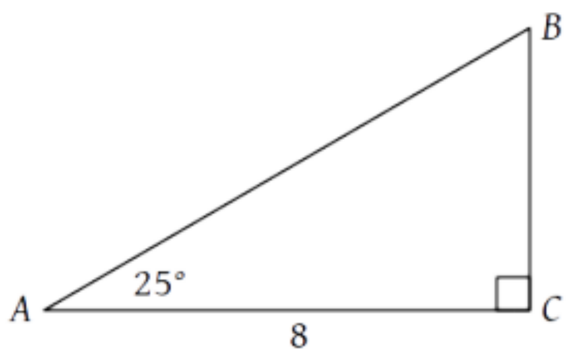
Specific trigonometric ratio corresponds to a *unique* angle measurement. For example, $1^\circ$ corresponds to a sine ratio of approximately 0.0175.

Using a scientific calculator, complete the following table:

| degrees | sine | cosine | tangent |
|---|---|---|---|
| 1 | | | |
| 10 | | | |
| 20 | | | |
| 30 | | | |
| 45 | | | |
| 60 | | | |
| 90 | | | |

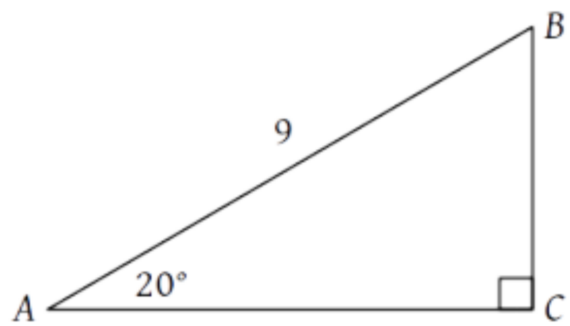
# Primary Trigonometric Ratios

Since the value of a trigonometric ratio relates directly to a unique ratio of sides, it is possible to solve for an unknown side by using this value. Consider the right triangle below. How can we solve for *a, c,* or angle B?

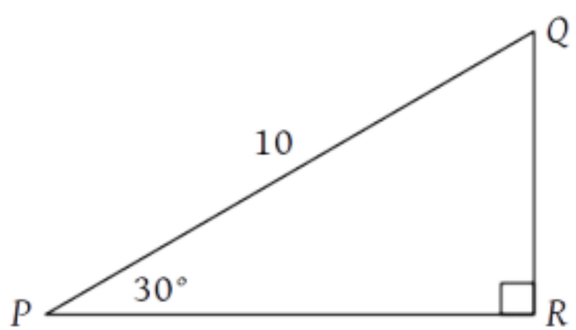


# Primary Trigonometric Ratios

**Example 1:** Determine $b$.

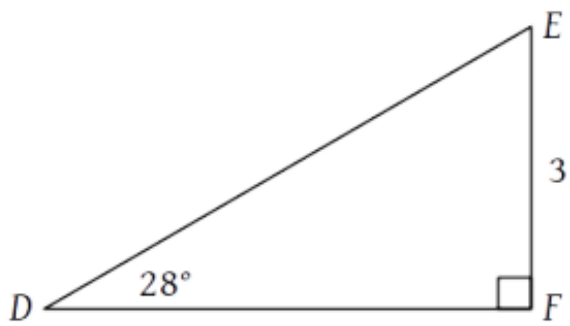



**Example 2:** Determine $p$.

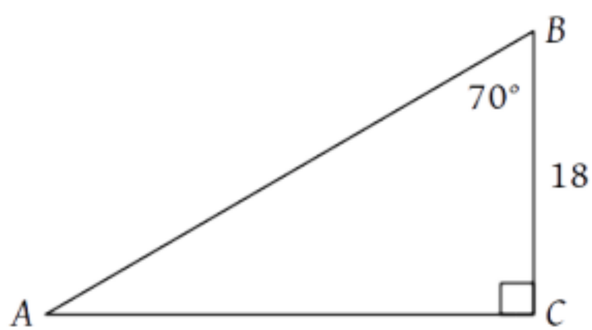


# Primary Trigonometric Ratios

**Example 3:** Determine $e$.

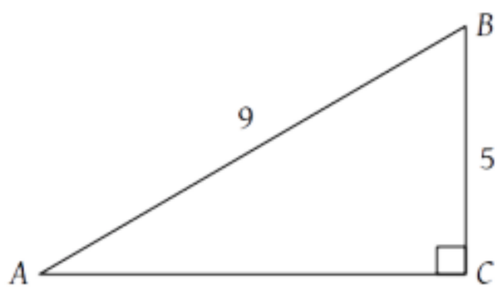



**Example 4:** Determine $c$.



# Using Trigonometric Ratios
# Part 2: Solving for Unknown Angles

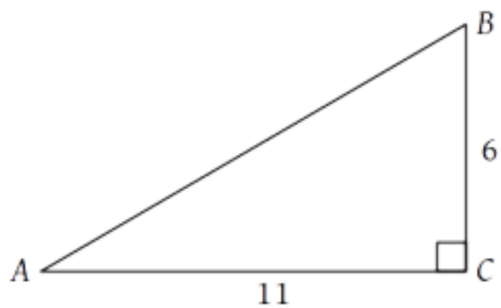
Consider the right triangle below. How can we find angle A?



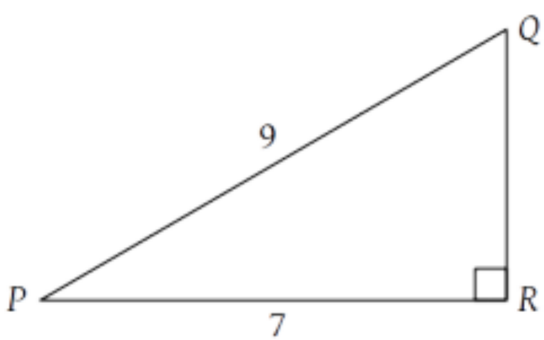

Using angle A as reference, we know both *a* (opposite) and *c* (hypotenuse), use the _____ ratio to relate the sides & angle.

# Primary Trigonometric Ratios

**Example 1:** Determine $\angle A$

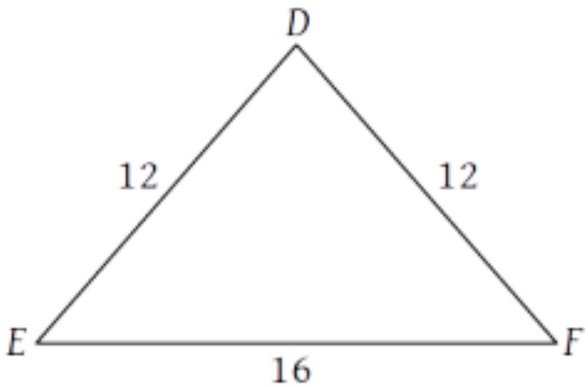



**Example 2:** Determine $\angle P$ and $\angle Q$



# Primary Trigonometric Ratios

**Example 3:** Determine $\angle D$



# Applications of Trigonometric Ratios

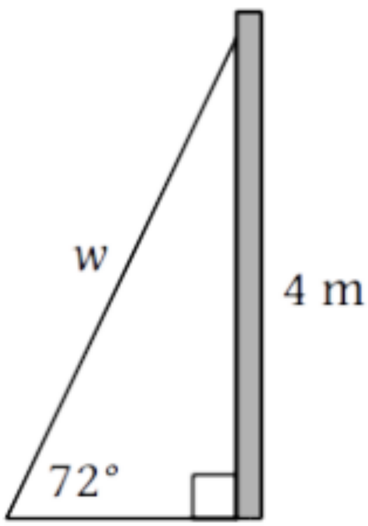
When solving **right angle** triangles, we use:

- Pythagorean Theorem (find a side, given 2 sides)
- Trigonometric ratios (find a side, given a side & an angle)
- Inverse trigonometric ratios (find an angle, given 2 sides)*

*Sometimes there is more than one way to solve a problem, but we always look for the most efficient method.

# Applications of Trigonometric Ratios

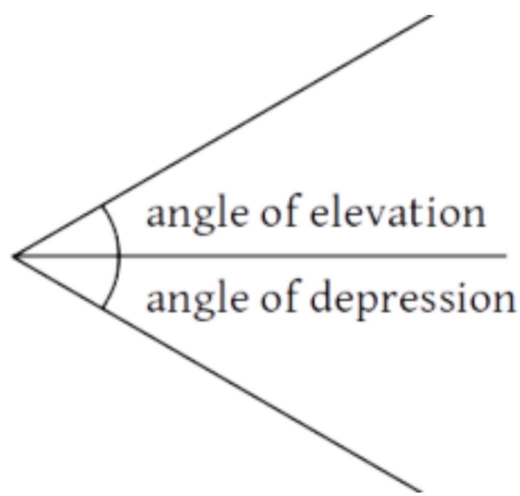
**Example 1**: A wire, attached 4 m up a telephone pole, makes an angle of 72° with the ground. How long is the wire?



# Applications of Trigonometric Ratios

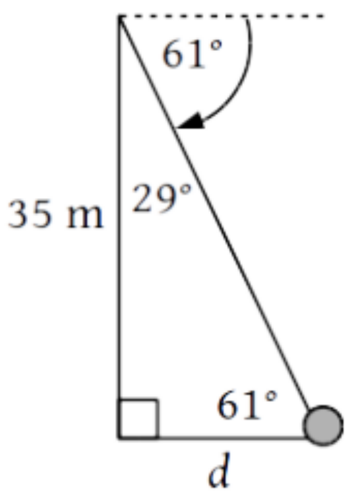
Sometimes an angle is specified in relation to another object, such as “10° with the ground” or “30° to the wall.

In other cases, an angle may be specified relative to a horizontal line. An angle measured upward from this line is an *angle of elevation,* while an angle measured downward is an *angle of depression*.



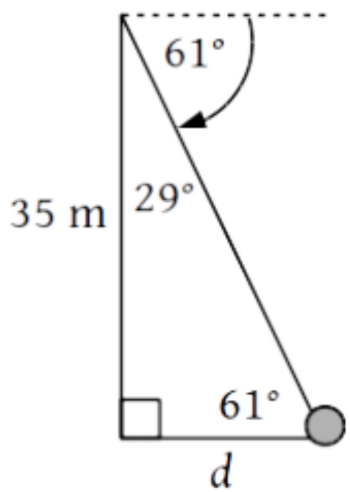
# Applications of Trigonometric Ratios

**Example 2**: From atop of a 35 m vertical cliff, the angle of depression to a large rock below is 61°. How far from the base of the cliff is the rock?



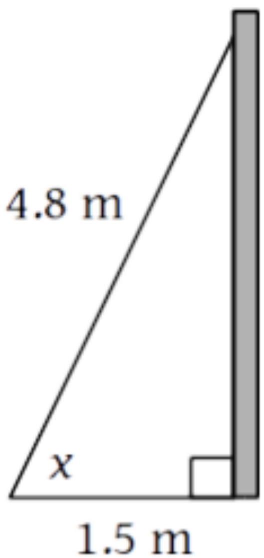
# Applications of Trigonometric Ratios

**Example 2**:



# Applications of Trigonometric Ratios

**Example 3**: The base of a 4.8 m ladder is placed 1.5 m from a wall. For safety reasons, the angle of elevation cannot exceed 67°. Is the ladder safe?



# Applications of Trigonometric Ratios

**Example 4**: Bart lives due east of Springville. Milhouse lives due south of the school, and is four times as far from school as Bart. If Bart and Milhouse live 3 km apart, how far from school does each person live?