Chapter 1: Introductions to Functions

§ 1.1: Functions and Relations

A function is a _______________ in which each value of the ___________ ____________

corresponds with **only one** value of the _______________ _________________.

Functions can be represented as:

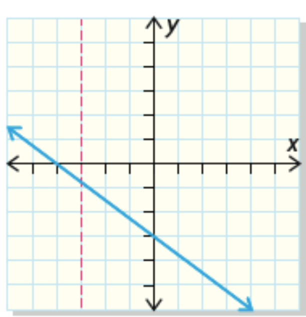
- a table of values
- a set of ordered pairs
- a map diagram
- a graph
- an equation.

____________: the set of all values of the independent variable (usually the x-values).

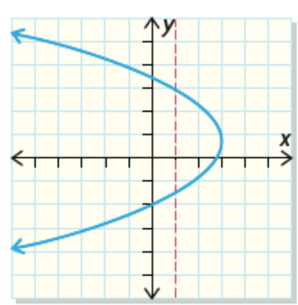
____________: the set of all values of the dependent variable (usually the y-values).

A graph represents a function if every vertical line intersects the graph in at most one point.

To check whether a graph represents a function, use the **vertical line test (VLT)**.



A relation that is a function



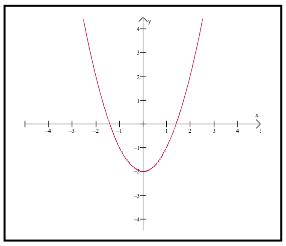
A relation that is not a function

You can also recognize whether a relation is a function from its <u>equation</u>.

- Linear relations (straight lines): $\boldsymbol{y = mx + b}$ or $\boldsymbol{Ax + By = C}$ are all functions.

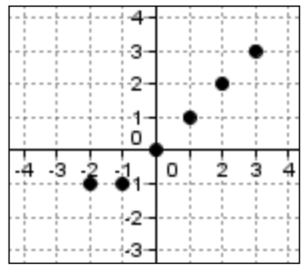
- Quadratics (parabolas) $\boldsymbol{y = ax^2 + bx + c}$ or $\boldsymbol{y = a(x-h)^2 + k}$ are also functions.

<u>Example 1:</u> Find the domain and range.

a) 

Domain: ________________________

Range: __________________________

b) 

Domain: ___________________________

Range: _____________________________

c) $\{(1, 3), (1, 4), (1, 5), (1, 6)\}$

Domain: ________________________

Range: __________________________

d) $y = \sqrt{x}$

Domain: ___________________________

Range: _____________________________

e) $y = \dfrac{1}{x}$

Domain: ________________________

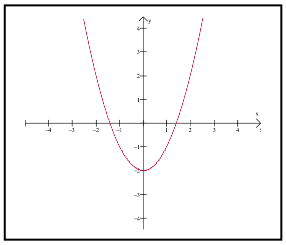
Range: __________________________

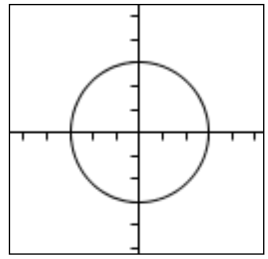
f) $y = (x-4)^2 - 3$

Domain: ___________________________

Range: _____________________________

Example 2: Which of the following are functions? Justify your answer.

a) 

b) 

c) (1, 3), (2, 4), (3, 5), (4, 6)}

d) {(1, 3), (1, 4), (1, 5), (1, 6)}

Mapping

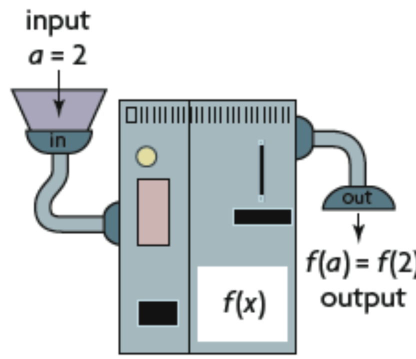
A mapping diagram is a representation that can be used when the relation is given as set of ordered pairs.

In Class Assignment: p. 10 #1, 2, 4; **Homework:** p. 11 #7, 8

§ 1.2: Function Notation

Symbols such as *f(x)* are called ____________ ____________, which is used to represent the value of the dependent variable *y* for a given value of the indepdent variable *x.* For this reason, *y* and *f(x)* are interchangeable in the equation of a function, so $y = f(x)$.

- *f(x)* is read "*f at x*" or "*f of x.*"
- *f(a)* represents the value or output of the function when the input is $x = a$. The output depends on the equation of the function.



To evaluate *f(a),* substitute *a* for *x* in the equation for *f(x).*

- *f(a)* is the *y*-coordinate of the point on the graph of *f* with *x*-coordinate *a.*

For example, if *f(x)* takes the value 3 at $x = 2$, then $f(2) = 3$ and the point (2, 3) lies on the graph of *f.*

<u>**Example 1:**</u>

If $f(x) = 2x + 3$, find:

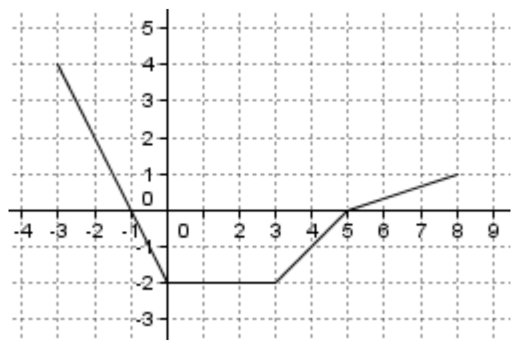
a) $f(6)$ b) $f(-5)$

A

c) $f(x+1)$ d) $f(2x)$

Example 2:

Given the graph to the right, find:



a) $f(2)$

b) $f(-3)$

c) x if $f(x)=2$

d) x if $f(x)=0$

Example 3:

A company rents cars for $50 per day plus $0.15/km.

a) Express the daily rental cost, C as a function of the number of kilometres, d travelled.

b) Determine the rental cost if you drive 472 km in one day.

c) Determine how far you can drive in a day for $80.

d) Is *C(d)* a function? Justify your answer.

In Class Assignment: p. 22 #1-3; **Homework:** pp. 23-24 5-7, 15, 16

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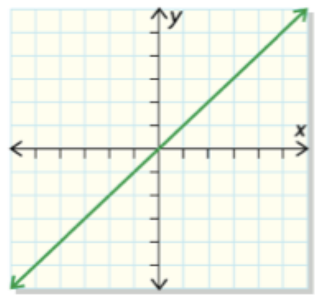
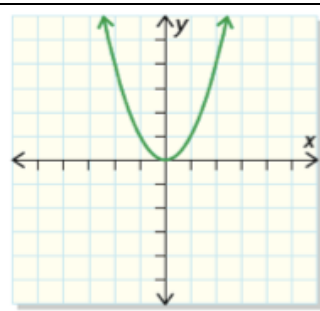
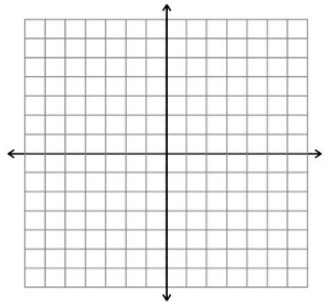
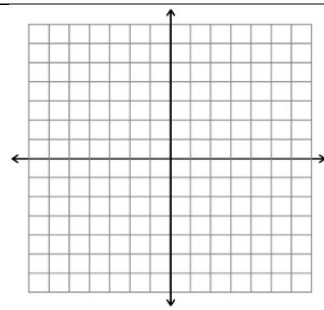
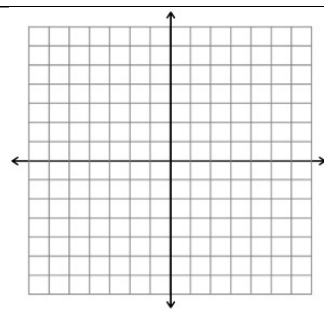
§ 1.3: Exploring Properties of Parent Functions

Equation of Function	**Name of Function**	**Sketch of Graph**	**Special Features/ Symmetry**	**Domain**	**Range**
$f(x) = x$	linear function		- straight line that goes through origin - slope is 1 - divides the plane exactly in half diagonally - graph only in quadrants 1 and 3		
$f(x) = x^2$	quadratic function		- parabola – opens up - vertex at the origin - y has a minimum value - y-axis is axis of symmetry - graph only in quadrants 1 and 2		

$f(x) = \sqrt{x}$	square root function				
$f(x) = \frac{1}{x}$	reciprocal function				
$f(x) = \lvert x \rvert$	absolute value function				

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§ 1.4: Determining the Domain and Range of a Function

The _____________ of a function is the set of all values of the independent variable of a relation for which the function is defined. The _____________ of a function depends on the equation of the function. The domain and range of a function can be determined from its ____________, from ________________________________, or from ________________________.

All linear functions include all the real numbers in their domains. **Real numbers** are numbers that are either ________________________ or ___________________________. These include positive and negative integers, zero, fractions, and irrational numbers such as $\sqrt{2}$ and π.

Linear functions of the form $f(x) = mx + b$, where $m \neq 0$, have range {_______}.

Consant functions $f(x) = b$ have range {_____}.

All quadratic functions have domain { __________ }. The range of a quadratic function depends on the ___________________ or ___________________ value and the

_______________________________________.

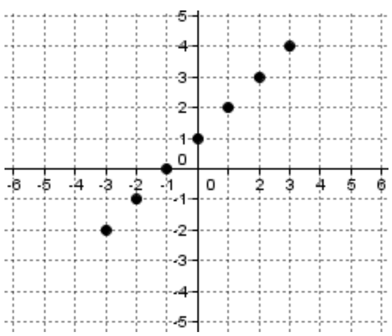
The domains of square root functions are _________________ because the square root sign refers to the _________________ square root. For example,

- The function $f(x) = \sqrt{x}$ has domain = { ____________ } and range = { ___________ }.
- The function $g(x) = \sqrt{x-3}$ has domain = { ___________ } and range = { _________ }.

When working with functions that model real-world situations, consider whether there are any restrictions on the variables. For example, negative values often have no meaning in a real context, so domain or range must be restricted must be restricted to nonnegative values.

Example: State the domain and range for the following.

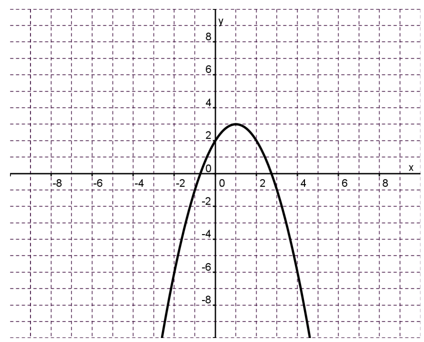
a)



Domain: ___________________________

Range: _____________________________

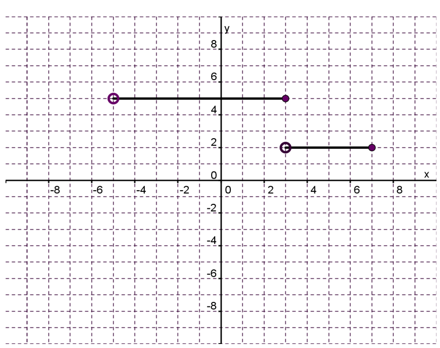
b)



Domain: ___________________________

Range: _____________________________

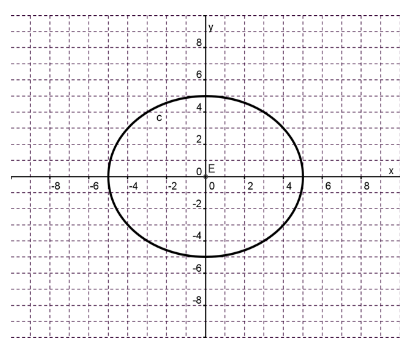
c)



Domain: ___________________________

Range: _____________________________

d)



Domain: ___________________________

Range: _____________________________

In Class Assignment: p. 35 #1-3, **Homework:** pp. 36-37 #6, 7, 9, 11

§ 1.5: The Inverse Functions and Its Properties

The inverse of a linear function is the reverse of the original function. It undoes what the original function has done. A way to determine the inverse function is to switch the two variables and solve for the previously independent variable.

- For example, if $y = 4x - 3$, rewrite this equation as $x = 4y - 3$ and solve for y to get $y = \frac{x+3}{4}$.
- f^{-1} is the notation for the inverse function of f.

<u>**Example 1:**</u> Determine the inverse of each function

a) $f(x) = 5x + 3$ b) $f(x) = -2$ c) $f(x) = 2x - 9$

<u>**Example 2:**</u> For each of the above determine $f^{-1}(3)$.

a)

b)

c)

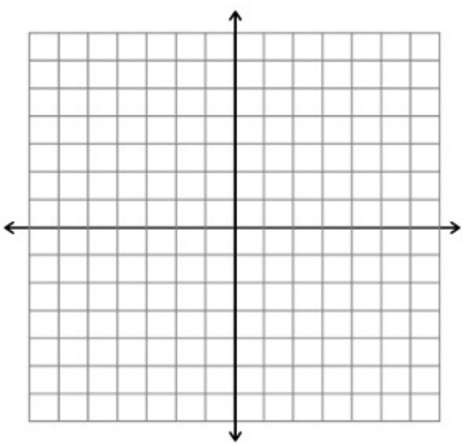
In Class Assignment: p. 48 #1, 2, 4; **Homework:** p. 49 #5, 9, 10

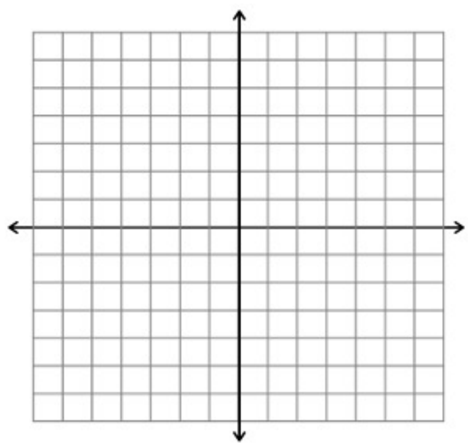
§ 1.6: Exploring Transformations of Parent Functions

In functions of the form $g(x) = af(x - d) + c$, the constants a, c, and d each change the location or shape of the graph of ***f(x)***. The shape of the graph of $g(x)$ depends on the graph of the parent function $g(x)$ and on the value of a.

Vertical Translations

Graph the parent function $f_1(x) = x^2$, then graph $f_2(x) = x^2 + 2$.

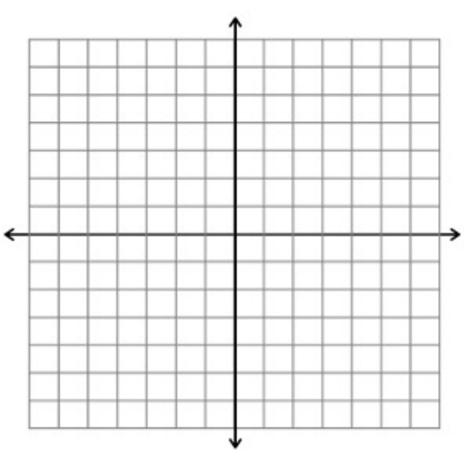


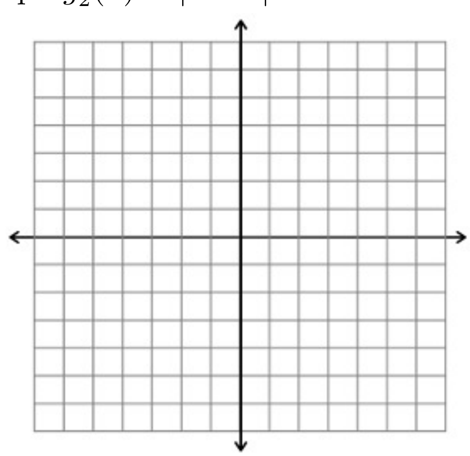


How does it compare? ____

Horizontal Translations

Graph the parent function $g_1(x) = |x|$, then graph $g_2(x) = |x - 3|$.

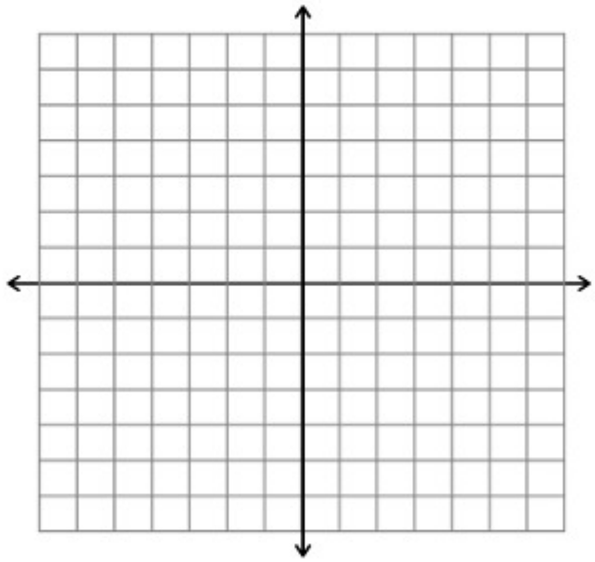


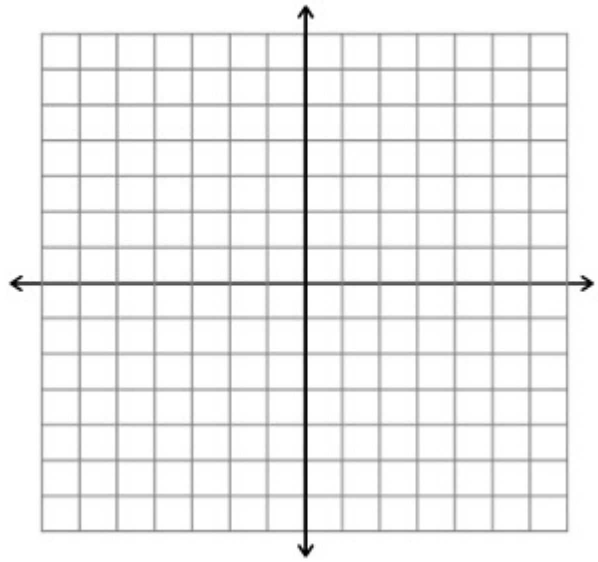


How does it compare? ____

Reflections

Graph the parent function $h_1(x) = \sqrt{x}$, then graph $h_2(x) = -\sqrt{x}$ and $h_3(x) = \sqrt{-x}$.





How does it compare? ____

In conclusion:

$\boldsymbol{f(x) \pm c}$ moves the graph ____________________________ by ________ units.

$\boldsymbol{g(x \pm d)}$ moves the graph ____________________________ by ________ units.

The graph of $-\boldsymbol{h(x)}$ is a reflection of the graph $\boldsymbol{h(x)}$ in ____________________________.

The graph of $\boldsymbol{h(-x)}$ is a reflection of the graph $\boldsymbol{h(x)}$ in ____________________________.

In Class Assignment: p. 51 #1, 2; **Homework:** p. 51 #3