Chapter 2: Equivalent Algebraic Expressions

§ 2.1: Adding and Subtracting Polynomials

Definitions:

Polynomial:

____

____

Like Terms:

____

____

Example 1:

a) $(3x^2 - 7x + 5) + (x^2 - 3x + 2)$

b) $(x^2 - 6x + 2) - (x^2 - 6x + 3)$

c) $2x(x + 1) + 3x(x - 1)$

d) $x(x - 6) - 2x^2(x - 4)$

Example 2:

Nyg and Petra are hosting a dinner for 300 guests. Cheers banquet hall has quoted these charges:

- \$500, plus \$10 per person, for food,
- \$200, plus \$20 per person, for drinks, and
- a discount of \$5 per person if the number of guests exceeds 200.

Nyg and Petra have created two different functions for the total cost, where n represents the number of guests and $n > 200$.

Nyg's cost function: $C_1(n) = (10n + 500) + (20n + 200) - 5n$

Petra's cost function: $C_2(n) = (10n + 20n - 5n) + (500 + 200)$

Are these functions equivalent?

In Class Assignment: p. 88 #1, 2, 3; Homework: p. 88 #6, 8

§ 2.2: Multiplying Polynomials

Suppose we want to simplify $f(x) \times g(x)$ where $f(x) = -5x + 2$ and $g(x) = -2x + 1$.

We can use the **3 Rules of Algebra:**

1. **The Commutative Property:** ____
2. **The Associative Property:** ____
3. **The Distributive Property:** ____

Example 1:

a) $2x(x + 5)$

b) $(2x + 1)(3x + 5)$

c) $(3x - 2)(x^2 - 5x + 1)$

d) $(2x - 3)^3$

Example 2:

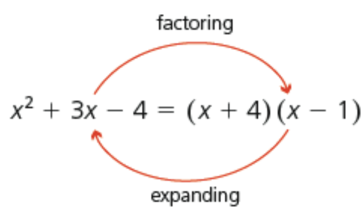
A rectangle is twice as long as it is wide. Predict how the area will change if the length of the rectangle is increased by 1 and the width is decreased by 1. Write an expression for the change in area and interpret the result.

In Class Assignment: p. 95 #1, 2, 3; Homework: p. 96 #4ace, 5bdf, 9, 11

Chapter 2: Equivalent Algebraic Expressions

§ 2.3: Factoring Polynomials

Factoring a polynomial means writing it as a ______. So factoring is the opposite of __________.



If a polynomial has more than three terms, you may factor it by ___________________. This is only possible if the grouping of terms allow you to divide the same ________________ _________________ from each group.

To factor a polynomial fully means that only ____ and ____ remain as a common factors in the factored expression. To factor polynomials fully, you can use factoring strategies that include:

- ____
- ____
- ____
- ____
- ____
- ____

Example 1: Common Factoring

a) $2x^2 + 4x$

b) $5y^3 + 15y^2$

c) $4x(x-5) + 3(x-5)$

d) $2y(3-y) - y(3-y)$

Example 2: Multi-Step Common Factoring aka Grouping

a) $f(x) = x^3 - x^2 - 2x - 2$

b) $g(x) = x^2 - y^2 - 10y - 25$

Example 3: Product and Sum

a) $x^2 - 4x - 12$

b) $y^2 + 6y + 8$

Example 4: Decomposition

a) $12x^2 - 7x - 10$

b) $2y^2 + 17y + 35$

Example 5: Difference of Squares

a) $x^2 - 25$

b) $100 - y^2$

c) $8x^2 - 50y^2$

In Class Assignment: p. 102 #1, 2, 3; Homework: p. 103 #4, 6, 7, 9, 10

§ 2.4: Simplifying Rational Functions

A rational function can be expressed as the __________ of two polynomial functions.

For example,

$$f(x) = \frac{6x+2}{x-1} \;;x \neq 1$$

Both rational functions and rational expressions are undefined for numbers that make the denominator ________________. Those numbers must be excluded or _____________ from being possible values for the variables. As a result, for all rational functions, the domain is _____________________________, except those numbers that make the denominator equal zero.

Rational functions and rational expressions can be simplified by factoring the ________________ and ____________ and then dividing both by their ____________ _______________ ____________.

The restrictions are found by determining all the _________ of the denominator. If the denominator contains two or more terms, the zeros can be determined from its _____________ _____________ before the function or expression is _________________.

<u>Example 1: Evaluate the following rational expressions for $x = 2$.</u>

a) $\frac{(x-3)}{x}$

b) $\frac{x-2}{x-5}$

c) $\frac{2x}{x-2}$

*Rational expressions are undefined when the denominator is zero.

Example 2: For which value(s) are the following rational expressions undefined?

a) $\frac{x^2-4x}{x+2}$

b) $\frac{x^2}{x+1}$

c) $\frac{3x}{x^2+9x+18}$

Example 3: Simplifying Rational Expressions

a) $\frac{2x^2-6x-36}{2x-12}$

b) $\frac{x^2-5x-6}{x^2-36}$

c) $\frac{2x^2-10x}{4x}$

d) $\frac{5-x}{x-5}$

In Class Assignment: p. 112 #1, 2, 3; Homework: p. 113 #4, 5, 7, 14

§ 2.5: Exploring Graphs of Rational Functions

The restricted values of rational functions correspond to two different kinds of graphical features:

____________________ and _______________________.

Holes occur at ________________ ______________ that result from a factor of the denominator

that is also a factor of the numerator. For example,

$$h(x) = \frac{x^2 + 7x + 12}{x + 3}$$

has a hole at ______________, since $h(x)$ can be simplified to the polynomial

$$h(x) = \frac{(x + 4)(x + 3)}{(x + 3)} = x + 4$$

Vertical asymptotes occur at restricted values that are still __________ of the denominator after

simplification. For example,

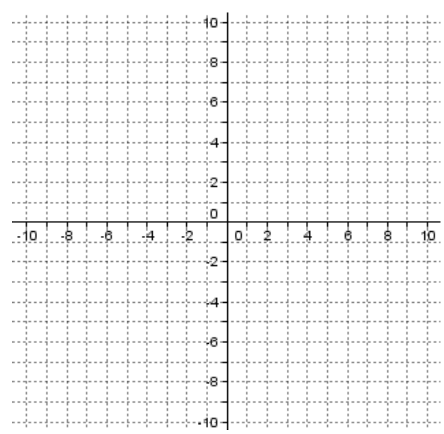
$$v(x) = \frac{5}{x - 8}$$

has a vertical asymptote at ___________.

<u>**Example 1: Holes**</u>

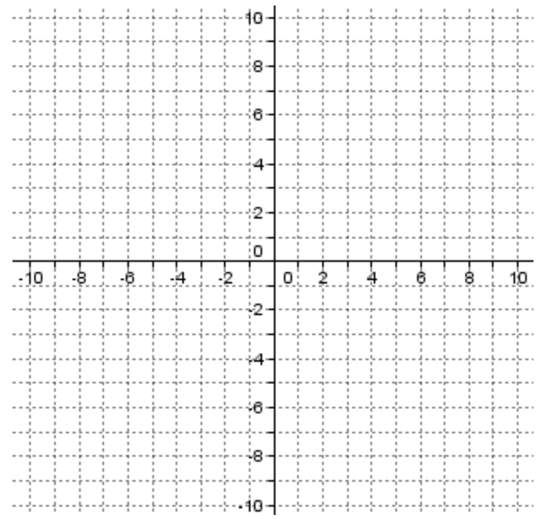
Some rational functions can be simplified to polynomials.

a) Graph the function: $f(x) = \frac{x^2-4}{x-2}$ then simplify:



c) Trace the graph near $x = 2$. Describe what happened to the graph at $x = 2$.

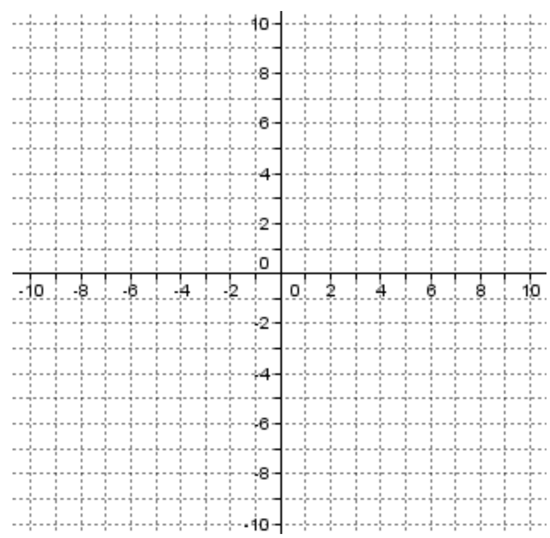
Example 2:



a) Determine another rational function that simplifies to a polynomial with a hole at $x = 1$. (i.e. restriction $x \neq 1$).

b) Graph your function and describe what happens to the graph at $x = 1$.

Example 3: Asymptotes

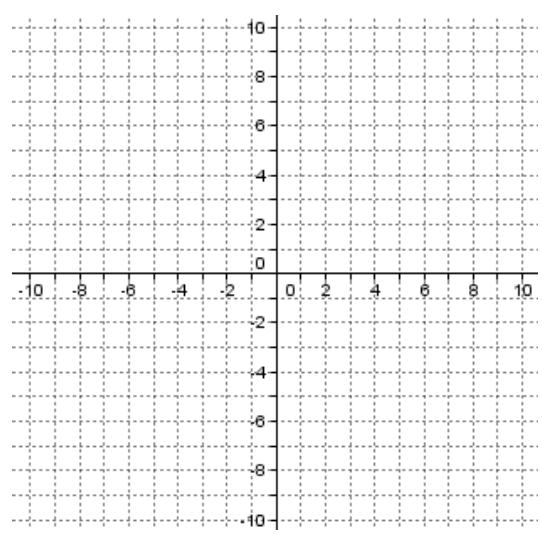


Some rational functions cannot be simplified.

a) Graph $g(x) = \frac{1}{x-2}$ and trace the graph near $x = 2$.

b) Describe what happens to the graph near $x = 2$.

Example 4:



a) Determine another rational function with a vertical asymptote at $x = 1$.

b) Graph your function and describe what happens to the graph at $x = 1$.

Homework: p. 116 #1-3

§ 2.6: Multiplying and Dividing Rational Functions

The procedures you use to multiply or divide rational numbers can be used to multiply and

divide ________________ _________________. That is, if *A, B, C,* and *D* are polynomials, then:

$$\frac{A}{B} \times \frac{C}{D} = \frac{AC}{BD}, \textit{provided that } \mathrm{B, D} \neq 0$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C} = \frac{AD}{BC}, \textit{provided that } \mathrm{B, D} \neq 0$$

To multiply rational expressions,

- _______________ the numerators and denominators, if possible
- _______________ out any factors that are common to the numerator and denominator.
- multiply the numerators, multiply the denominators, and then write the result as a

  _____________ rational expression

To divide two rational expressions,

- factor the numerators and denominators, if possible
- multiply by the _______________ of the divisor
- divide out any factors ________________ to the numerator and denominator
- write the result as a single rational expression

To determine the _______________, solve for the zeros of all of the denominators in the factored expression, in the case of division, bot the numerator and denominator of the divisor must be used. Both are needed because the _____________ of this expression is used in the calculation.

Example 1: Simplify

a) $\frac{2x}{5} \times \frac{5x}{2}$

b) $\frac{(x+2)(x-2)}{(x-4)} \times \frac{(x-4)}{2(x-2)}$

Example 2: Simplify and State Restrictions

a) $\frac{(x+2)^2}{(x-2)^2} \times \frac{x^2-4x+4}{2(x+2)}$

b) $\frac{21x-3x^2}{16x+4x^2} \div \frac{14x-9x+x^2}{12+7x+x^2}$

In Class Assignment: p. 121 #1, 3, 4; Homework: p. 122 #5cd, 6ac, 7bd, 8