Appendix A Review of Prerequisite Skills

Evaluating expressions

To evaluate the expression $2x^2 - 7xy + 5$ for $x = 2$ and $y = -3$, substitute 2 for x and -3 for y in the expression. Then, simplify using the order of operations.

$$\begin{aligned} 2x^2 - 7xy + 5 &= 2(2)^2 - 7(2)(-3) + 5 \\ &= 8 + 42 + 5 \\ &= 55 \end{aligned}$$

1. Evaluate for $x = -2$, $y = 5$, and $z = 4$.

a) $x^2 + 5x - y^2$ **b)** $(2x + y)(3z - 2y)$ **c)** $8xy + 3y^3 - 6z$

d) $-5y - 4x^2y^2 + 3$ **e)** $2z - y(3x^2 - 4y)$ **f)** $8 + 6yx - 7y^2$

g) $xyz - xy - xz - yz$ **h)** $5x^2 - 9z^2y + 1$ **i)** $(xz - xy)(xz + xy)$

j) $\dfrac{(x-y)(x+z)}{(y+z)^2}$ **k)** $\dfrac{(x+1)(y-3)}{(x-6)(z+1)}$ **l)** $\dfrac{(x-5)^2(y-3)^2(z+1)^2}{(x-y+z+4)^3}$

Exponent laws

The exponent laws are used to simplify the following expression.

$$\frac{(3x^2y)^3(8x^5y^7)}{2x^4y^9}$$

The exponent 3 is shared by all parts inside the brackets:

$$= \frac{(27x^6y^3)(8x^5y^7)}{2x^4y^9}$$

Multiply the numeric coefficients and add exponents on like variables:

$$= \frac{216x^{11}y^{10}}{2x^4y^9}$$

Divide the numeric coefficients and subtract exponents on like variables:

$$= 108x^7y$$

1. Use the exponent laws to simplify each of the following.

a) $(4x^2)^5$ **b)** $(-3x^4y^5)^2$ **c)** $5(2x^3y)(5x^2y^2)^3$

d) $\dfrac{1}{9}\left(\dfrac{2}{5}\right)^2\left(\dfrac{3}{4}\right)^3$ **e)** $\left(\dfrac{a^2}{b}\right)^5\left(\dfrac{b^2}{a}\right)^3$ **f)** $\dfrac{(9x^2y^4)(-15x^5y^7)}{(3xy)(5x^2y^6)}$

g) $\left(\dfrac{2}{x^2}\right)^5(3x^2y^3)(4x^3)^2$ **h)** $\dfrac{(-2x^3y^3)^3(5x^2y)^4}{(10xy^3)^2}$

Fractions, percents, decimals

The following table shows how equivalent fractions, percents, and decimals can be expressed.

Fraction	Percent	Decimal
$\frac{63}{100}$	63%	0.63
$\frac{8}{100} = \frac{2}{25}$	8%	0.08
$\frac{0.5}{100} = \frac{1}{200}$	0.5%	0.005
$\frac{150}{100} = \frac{3}{2}$	150%	1.5

1. Copy and complete the following table. Express all fractions in their simplest form.

	Fraction	Percent	Decimal
a)	$\frac{75}{100}$		
b)	$\frac{1}{2}$		
c)	$8\frac{2}{5}$		
d)		34%	
e)		0.03%	
f)		5.6%	
g)			0.45
h)			0.03
i)			2.68

2. Find the percent change when the price of gasoline jumps from $0.699/L to $0.799/L.

3. A $25-shirt is on sale for 20% off. What is the total cost of the shirt, including 8% PST and 7% GST?

Graphing data

The table gives the ages of 80 cars sold at a used car lot.

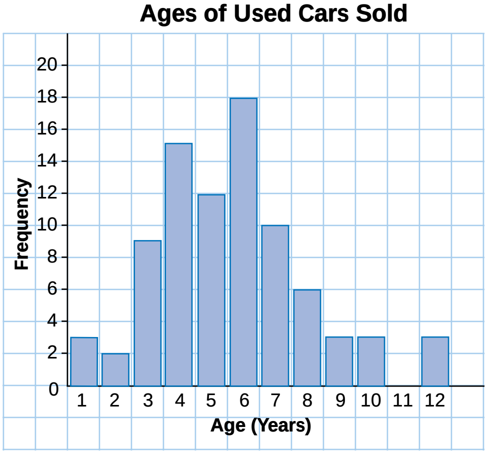
Age (years)	Frequency	Percent
1	3	3.75
2	2	2.5
3	7	8.75
4	15	18.75
5	12	15
6	18	22.5
7	10	12.5
8	6	7.5
9	3	3.75
10	3	3.75
11	0	0
12	1	1.25

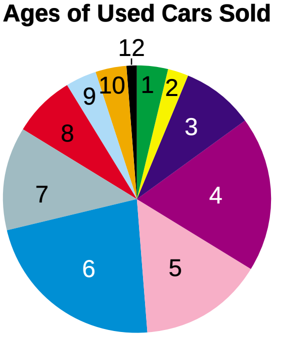
The percents were calculated by dividing each frequency by 80 and multiplying by 100.

The most frequent age of cars sold was 6 years old, at 22.5% of the total.

The least frequent age was 11 years old, at 0% of the total.

These data can be graphed using a bar graph or a circle graph as shown below.





1. The table shows the number of tickets sold for a two-week theatre production of *Little Shop of Horrors*.

a) Construct a line graph of the ticket sales.

b) Describe the trend in ticket sales.

c) What types of graphs would not be suitable for these data?

Day	Tickets Sold
1	2350
2	2350
3	2350
4	2350
5	2189
6	2012
7	1850
8	1878
9	1504
10	920
11	1267
12	1422
13	998
14	835

2. Grade 12 students were asked to provide the number of universities to which they were considering applying. The following results were obtained.

Number of Universities	Frequency
0	35
1	25
2	56
3	27
4	18
5	12
6	3

a) How many students were surveyed?

b) Construct a bar graph.

c) Determine the percent of total responses for each number.

d) Construct a circle graph.

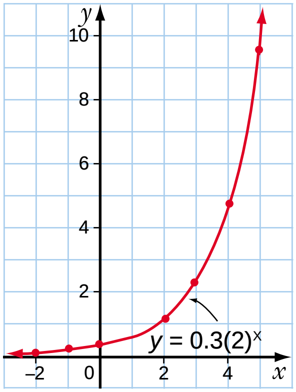
e) What type of graph would not be suitable for these data?

Graphing exponential functions

In the exponential function $y = 0.3(2)^x$, the base is 2, the numerical coefficient is 0.3, and the exponent is x. Complete a table of values by finding the value of y for each value of x.

Plot the points on a grid and draw a smooth curve through the points.

x	y
–2	0.075
–1	0.15
0	0.3
1	0.6
2	1.2
3	2.4
4	4.8
5	9.6



1. Graph each of the following functions.

a) $y = 2^x$

b) $y = 2^{-x}$

c) $y = 1.5(2)^x$

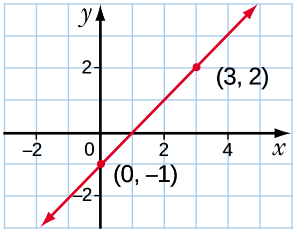
d) $y = 3(0.5)^x$

e) $y = 0.1(4)^x$

f) $y = 81\left(\frac{1}{3}\right)^x$

Graphing linear equations

To graph the line $y = \frac{2}{3}x - 1$ using the slope and y-intercept form $y = mx + b$, identify the slope as $m = \frac{2}{3}$ and the y-intercept as $b = -1$. Plot the y-intercept first. Then, plot a second point by moving 3 to the right (run) and 2 upward (rise) to the point (3, 2).



1. Graph each of the following using the slope and y-intercept.

a) $y = \frac{5}{3}x + 2$

b) $y = 4x - 5$

c) $y = -5x + 3$

d) $y = -\frac{1}{2}x + 2$

e) $y = -x - 2$

f) $y = 6x - 7$

g) $2x + y = 8$

h) $3x + 4y = 10$

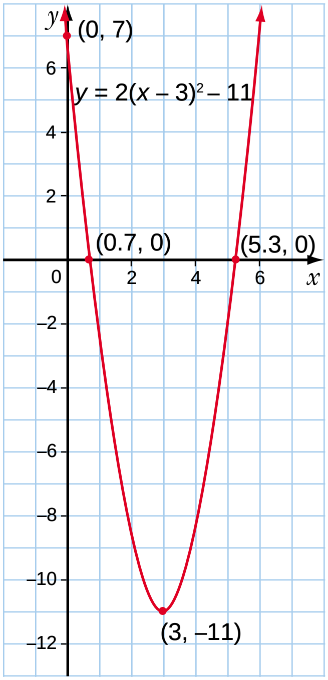
i) $5x - 6y = 9$

Graphing quadratic functions

To graph the function $y = 2x^2 - 12x + 7$, you must first complete the square.

	$y = 2x^2 - 12x + 7$
Factor 2 from the x^2 and x terms:	$y = 2(x^2 - 6x) + 7$
Add and subtract the square of half of the coefficient of x:	$y = 2(x^2 - 6x + 3^2 - 3^2) + 7$
	$y = 2(x^2 - 6x + 9) - 2(9) + 7$
Complete the square:	$y = 2(x - 3)^2 - 11$

To graph $y = 2(x - 3)^2 - 11$, plot the vertex at $(3, -11)$.
Sketch the parabola opening upward, stretched vertically a factor of 2.
To find the y-intercept, substitute $x = 0$ and evaluate $y = 7$.
To find the x-intercepts, substitute $y = 0$ and evaluate $x \doteq 5.3$ or $x \doteq 0.7$.



1. Graph the following functions and estimate any x- and y-intercepts.

a) $y = 3x^2$ **b)** $y = -3x^2$ **c)** $y = 2(x - 1)^2 + 3$

d) $y = 3x^2 + 6x - 5$ **e)** $y = -2x^2 + 8x + 10$ **f)** $y = 5x^2 - 8x + 2$

g) $y = x^2 + 6x - 9$ **h)** $y = -x^2 + 5x + 3$ **i)** $y = 4x^2 + 6x + 1$

Mean, median, mode

The following marks were scored on a test marked out of 50.

45 38 26 44 45 20 38 32 29 18 32 33 33 41 45 50 31 27 25 24
38 36 25 35 42 30 38 31 32 29 30 39 38 39 25 26 21 49 12 34

The marks can be organized into a stem-and-leaf plot.

Test Marks	
1	2, 8
2	0, 1, 4, 5, 5, 5, 6, 6, 7, 9, 9
3	0, 0, 1, 1, 2, 2, 2, 3, 3, 4, 5, 6, 8, 8, 8, 8, 8, 9, 9
4	1, 2, 4, 5, 5, 5, 9
5	0

The mean is the sum of all the measures, divided by the number of measures.

$$\text{mean} = \frac{1325}{40}$$
$$= 33.125$$

The median is the middle measure when all the measures are placed in order from least to greatest.

$\frac{1}{2}$ of 40 = 20

The median is the midpoint of the 20th and 21st measures.

$$\frac{(32 + 33)}{2} = 32.5$$

The mode is the most frequent measure.
mode = 38

1. Calculate the mean, median, and mode for each of the following sets of data.

a) Student ages in years:
18 16 13 15 16 18 18 18 15 17 17 18 14 15 16 18 19 20 19
16 14 17 17 18 15 16 18

b) Prices, in dollars, of the Mario Lemieux rookie card at various stores:
89 58 79 79 47 99 88 125 79 89 64 79 78 90 95 75 89 79 79

Number patterns

A pattern can be described by identifying the operation that is needed to find successive terms.

44, 33, 22, . . . is a sequence of terms found by subtracting 11.

The next three terms would be 11, 0, –11.

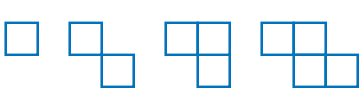
1. Describe each of the following patterns. Find the next three terms.

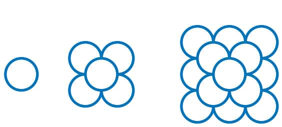
a) 15, 12, 9 **b)** 4, 12, 36 **c)** 1, 4, 9

d) 2, 4, 6 **e)** 9, –3, 1 **f)** 80, –40, 20

g) 1, –1, 1, –1 **h)** 15, 5, –5 **i)** 1, 2, 4, 8, 16

j) $\frac{3}{4}, \frac{1}{2}, \frac{1}{3}$ **k)** p, pq, pqq, pqqq **l)** a, ab, abb, abbbb

m)


n)


Order of operations

To evaluate this expression, use the order of operations to multiply first and then simplify by subtracting.

$$
\begin{aligned}
(-2)(-6) - (5)(-3)^2 &= 12 - (5)(9) \\
&= 12 - 45 \\
&= -33
\end{aligned}
$$

1. Evaluate each expression.

a) $(-3)(5) + (-6)(-8)$

b) $(12)(11) - (-9)(7)$

c) $-10(3)(2) + (-12)(7)(-2)$

d) $0.3(5.5)^2 - (-6.7)(2.1)^3 + 4.2(-1.1)^5$

e) $3.2\left(\frac{1}{2}\right) - 2.5\left(\frac{3}{5}\right) + 1.4\left(\frac{2}{7}\right)$

f) $\left(\frac{3}{4}\right)^2\left(\frac{8}{3}\right)$

g) $\frac{5}{2} - \frac{2}{5}\left(\frac{4}{3}\right)^3$

h) $\frac{2}{3} - \frac{1}{2}\left(\frac{5}{6} - \frac{7}{4}\right)$

Ratios of areas

One rectangle has dimensions of 6 cm by 7 cm. A second rectangle has dimensions that are 3 times those of the first. The ratio of their areas can be found in two ways.

Method 1: The area of the first rectangle is $6 \times 7 = 42$ cm^2.
The area of the second rectangle is $18 \times 21 = 378$ cm^2.
The ratio of their areas is 42:378 = 1:9.

Method 2: The dimensions have a ratio of 1:3.
The ratio of the areas is the square of the ratio of the dimensions.
So, the ratio is $1^2:3^2 = 1:9$.

1. a) A triangle has base of 15 mm and height of 8 mm. The dimensions of a second triangle are double those of the first. What is the ratio of their areas?

b) The dimensions of a square are triple those of another square. What is the ratio of the areas of the two squares?

c) The length and the width of a parallelogram are halved to create a second parallelogram. What is the ratio of the areas of the two parallelograms?

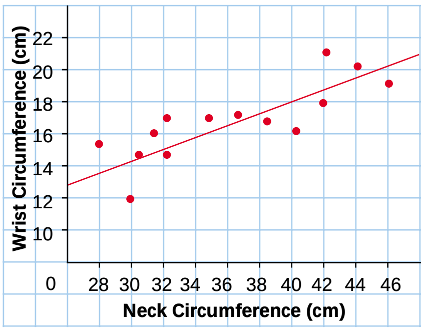
d) A pyramid has edges that are five times as long as the edges of another pyramid. What is the ratio of the volumes of the two pyramids?

Scatter plots

The measurements of the neck and wrist circumferences, in centimetres, of fifteen 17-year-old students, are recorded in the following table.

Neck	32	31	44	36	29	34	40	38	42	32	33	35	40	39	30
Wrist	15	15	19	17	15	16	21	17	20	16	17	17	18	17	12

Plot the data in a scatter plot, showing the linear relationship between the two measurements. Construct a line of best fit through the data.



Refer to Appendix B: Technology for help in constructing the scatter plot and line of best fit using technology.

The slope of the line of best fit is positive.

Half of the points are above the line of best fit and the other half of the points are below the line.

The line can be extrapolated to find the wrist circumference of a 17-year-old person with a neck circumference of greater than 44 cm or less than 29 cm. A person with a neck circumference of 47 cm would expect to have a wrist circumference of about 21 cm. A person with a neck circumference of 28 cm would expect to have a wrist circumference of about 14 cm.

1. The following table shows the temperature of Earth's atmosphere at various altitudes on a particular day.

a) Construct a scatter plot, with Altitude along the horizontal axis and Temperature along the vertical axis.

b) Describe the relationship between altitude and temperature.

c) Draw a line of best fit through the data. Describe your steps.

d) What is the slope of your line of best fit? What does the slope represent?

e) Write an equation relating temperature to altitude.

f) What would the temperature be at an altitude of 12 km?

Altitude (km)	Temperature (°C)
0	13
1	0
2	–1
4	–13
5	–20
6	–27
8	–41
10	–55

2. In the city of Marktown, the annual cost, in dollars, of heating a house relative to its floor space, in square metres, is shown in the following table.

Floor Space (m^2)	Heating Cost ($)
140	1071
160	1279
180	1452
200	1599
220	1821
240	2002
260	2242
280	2440

a) Construct a scatter plot, with Floor Space along the horizontal axis and Heating Cost along the vertical axis.

b) Describe the relationship between floor space and heating cost.

c) Draw a line of best fit through the data. Describe your steps.

d) What is the slope of your line of best fit? What does it represent?

e) Write an equation relating floor space and heating cost.

f) What would the heating cost be for a floor space of 350 m^2?

Sigma notation

The following series can be written in sigma notation by writing a formula representing the terms and by writing the lower and upper limits of the variable, i, as 1 and 12, respectively.

$$1^2 + 2^2 + 3^2 + \ldots + 12^2 = \sum_{i=1}^{12} i^2$$

1. Express the following series in sigma notation.

a) $2 + 4 + 8 + 16 + 32 + 64$

b) $3! + 4! + 5! + \ldots + 15!$

c) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{8}$

d) $5x + 6x + 7x + 8x + 9x + 10x + 11x$

2. Expand each of the following. Do not simplify.

a) $\sum_{i=1}^{10} 3i$

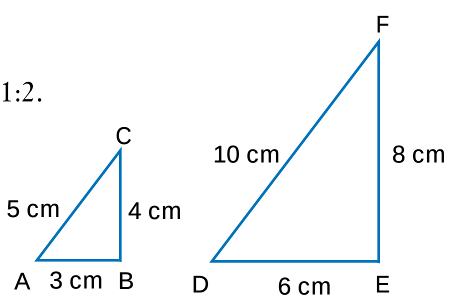
b) $\sum_{m=1}^{8} 5(2)^m$

c) $\sum_{i=4}^{15} (2i - 1)$

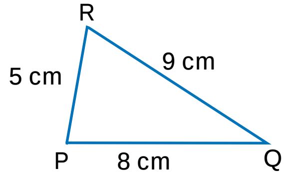
d) $\sum_{k=3}^{6} \frac{k+1}{k^2}$

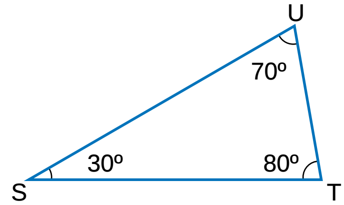
Similar triangles

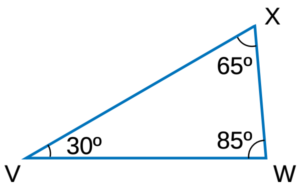
In similar triangles, corresponding angles are equal and corresponding sides are proportional.
$\triangle ABC \sim \triangle DEF$ because AB:DE = BC:EF = CA:FD = 1:2.

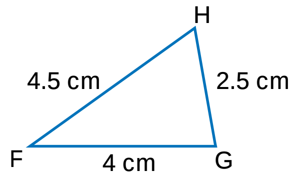


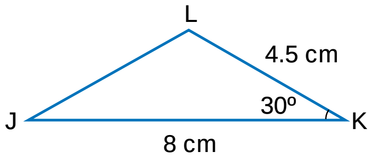
1. Determine which of the following triangles are similar. Explain.

a) 

b) 

c) 

d) 

e) 

Simplifying expressions

The expression $(x - y)(2x - 3y)^2$ can be simplified by expanding the brackets and then collecting like terms.

$$\begin{aligned}(x - y)(2x - 3y)^2 &= (x - y)(4x^2 - 12xy + 9y^2)\\ &= 4x^3 - 12x^2y + 9xy^2 - 4x^2y + 12xy^2 - 9y^3\\ &= 4x^3 - 16x^2y + 21xy^2 - 9y^3\end{aligned}$$

1. Expand and simplify.

a) $(x + 2y)^2$ b) $(2x - 5)^2$ c) $(x^2 + y)^2$

d) $(5x - 2y)^2$ e) $(3x + 4)(2x + 1)^2$ f) $(5y - 2)^2(3y + 4)$

g) $(k - 3)^2(k + 4)$ h) $(5m - n)^2(5m + 2n)$ i) $(2y - 4x)(5y + 4x)^2$

Solving equations

To solve this equation, expand the brackets, then collect like terms on one side of the equation.

$$\begin{aligned}5(x + 3) &= 4(x + 7)\\ 5x + 15 &= 4x + 28\\ 5x - 4x &= 28 - 15\\ x &= 13\end{aligned}$$

To solve an equation involving exponents, take the root of each side.

$$\begin{aligned}x^2 &= 64\\ x &= \pm\sqrt{64}\\ x &= \pm 8\end{aligned}$$

1. Solve for x.

a) $4x - 5 = 3x - 9$
b) $4x + 3 = 2x - 9$
c) $7x + 6 = 2x - 9$
d) $8(x - 3) = 3(2x + 4)$
e) $\frac{3}{4} = \frac{c}{10}$
f) $\frac{4.8}{1.2} = \frac{6.3}{y}$
g) $\frac{3x - 2}{5} = \frac{2x + 1}{3}$
h) $\frac{5x - 2}{4} = \frac{3x + 1}{2}$
i) $x^2 = 36$
j) $x^2 = 144$
k) $x^3 = 8$
l) $x^3 = 216$

Substituting into equations

To simplify $f(g(x))$, substitute $g(x)$ for x in $f(x)$, expand each set of brackets and simplify by collecting like terms.

Given $f(x) = 2x^2 - 3x + 1$ and $g(x) = 5x + 4$.

$$\begin{aligned} f(g(x)) &= 2(5x + 4)^2 - 3(5x + 4) + 1 \\ &= 2(25x^2 + 40x + 16) - 15x - 12 + 1 \\ &= 50x^2 + 80x + 32 - 15x - 12 + 1 \\ &= 50x^2 + 65x + 21 \end{aligned}$$

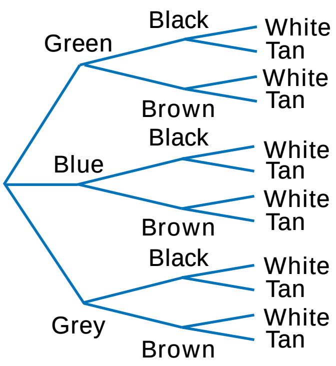
1. Given $f(x) = 3x^2 - 6$, $g(x) = x^2 + 2$, and $h(x) = 2x + 1$. Substitute and simplify.

a) $f(-1)$
b) $g(5)$
c) $h(-10)$
d) $g(f(x))$
e) $g(f(-1))$
f) $f(g(x))$
g) $f(g(4))$
h) $h(f(x))$
i) $f(f(x))$
j) $f(h(2))$
k) $g(g(x))$
l) $g(g(10))$
m) $h(g(-1))$
n) $f(g(h(3)))$

2. In general, are $f(g(x))$ and $g(f(x))$ equal? Explain.

Tree diagrams

The tree diagram on the right illustrates the number of ways to pick an outfit from a green, blue, or grey shirt, black or brown pants, and a white or tan jacket. Each new decision branches out from the previous one to build a tree diagram.



1. A spinner has four colours—red, green, orange, and purple. Draw a tree diagram to illustrate the possible outcomes for two spins.
2. A hockey team played three games in a tournament. The results could be a win, loss, or tie. Draw a tree diagram to illustrate the possible outcomes in the tournament.
3. A family is planning a vacation. They could fly, drive, take the train, or take the bus from Toronto to Sudbury. They could drive or take the bus from Sudbury to Timmins. They could drive, take the train, or fly from Timmins to Winnipeg. Draw a tree diagram to illustrate the possible ways of taking their vacation.